

Effect of Symmetric and Anti-symmetric Horizontal Ionospheric Structure on Oblique Propagation

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Abstract

The structure of the ionosphere can be represented, in general, in terms of the ionosphere at the mid-point of an oblique path, plus the symmetric and anti-symmetric parts of the deviation of the ionosphere from that at the mid-point. It is possible to draw general conclusions about the effect of the two parts on radio propagation over the path. These conclusions are valid to the first order, and depend upon magneto-ionic effects being negligible over the paths considered.

Perhaps the most important conclusion is that structure which is anti-symmetric about the mid-point does not affect the time of flight over the path (*i.e.* group path). In consequence, the maximum useable frequency is also unaffected. However the elevation angles at each end of the ray suffer equal and opposite changes. Ray paths may be lost because of obscuration by the earth. An example of structure which is anti-symmetric is an ionospheric tilt about the mid-point.

Structure that is symmetric generally does affect the group path and maximum useable frequency. There are also symmetric changes in the elevation angles. The ionosphere is approximately symmetric about the magnetic equator, and significant increases in maximum useable frequency are often observed in trans-equatorial propagation.

In the case where only anti-symmetric horizontal structure is present, the common practice of predicting the performance of oblique ionospheric radio paths using the mid-point ionosphere is justified. Changes in elevation and bearing can be simply estimated. However, when symmetric structure is present the common approach will lead to errors.

Introduction

This paper is concerned with oblique radio propagation between points on the earth via the ionosphere. In the medium and high frequency bands it is possible to model accurately the characteristics of such propagation using ray theory [1].

Ray tracing using a realistic model of the ionosphere requires considerable computer time which may be expensive, and even on super computers may be too slow in real-time applications. Frequently the ionospheric structure is not known sufficiently accurately to justify such an approach. For these reasons, ionospheric radio propagation is frequently

analysed, and systems designed, making use of the ionospheric profile at the mid-point of the path. Residual effects associated with horizontal ionospheric gradients are sometimes accounted for by the introduction of ionospheric tilts.

This paper considers the first order effect of such horizontal gradients, and finds that it is useful to distinguish between situations in which departures of the ionosphere from the profile at the mid-point are either symmetric or anti-symmetric about a vertical plane through the mid-point. Of course, any horizontal ionospheric structure can be represented as the sum of the symmetric and anti-symmetric parts.

The anisotropy associated with magneto-ionic effects is neglected throughout. This means the results apply in practice at frequencies large compared with the gyro-frequency, and on longer oblique paths. Some specific results are given in cartesian coordinates. While corresponding results can be obtained in spherical coordinates, there is no loss of generality since the transition from plane to spherically stratified ionosphere represents a symmetric variation for any path.

Variational formulation

The key to the method is to represent the departure of the ionospheric properties from those at the mid-point as a perturbation or variation to the medium. The theory of Bennett[2], which owes its origin to the work of Hamilton[3] may then be applied. Thus, the squared refractive index of the ionosphere may be written

$$\mu^2 = (\mu^2 - \mu_{m.p.}^2) \delta m + \mu_{m.p.}^2 \quad (1)$$

The actual situation, including horizontal structure, is then represented by setting $\delta m = 1$ while the situation neglecting horizontal structure is represented by $\delta m = 0$. We obtain results for the variations in the ray which are of the first order in δm . It is in this sense that the results are valid. It has been found by detailed calculation that the results apply to high accuracy in a number of practical cases.

Unperturbed situation

In Hamiltonian form, in the absence of horizontal gradients, the ray equations may be written

$$\begin{aligned} \frac{d}{du} p_x &= 0 & \frac{d}{du} x &= p_x \\ \frac{d}{du} p_y &= 0 & \frac{d}{du} y &= p_y \\ \frac{d}{du} p_z &= \frac{1}{2} \frac{\partial \mu_{m.p.}^2}{\partial z} & \frac{d}{du} z &= p_z \end{aligned} \quad (2)$$

$$\frac{p_x p_x + p_y p_y + p_z p_z}{\mu_{m.p.}^2(z)} - 1 = 0$$

It is assumed the plane of propagation (corresponding to the great-circle plane) is the plane $y = 0$, and that z is altitude. The variable of integration u is elapsed group path along the unperturbed ray. This choice of ray parameter offers advantages in both theoretical and numerical work.

From the equations (2) we can write

$$\begin{aligned}
p_x &= \text{const} & \frac{d}{du}x &= p_x \\
p_y &= \text{const} = 0 & y &= \text{const} = 0 \\
p_z &= \pm\sqrt{\mu_{m.p.}^2 - p_x^2} & \frac{d}{du}z &= p_z
\end{aligned}$$

It is clear from the equations (2) that, in the absence of horizontal gradients, the ray is symmetric about the midpoint (where $p_z = 0$) as expected. The phase path in the absence of horizontal gradients is given by

$$P = \int_A^B \mathbf{p} \cdot \frac{d}{du} \mathbf{x} du = \int_A^B \mu_{m.p.}^2 du \quad (3)$$

where A and B represent the end points of the ray.

Variation of the phase path and the group path

Introduction of horizontal gradients cause the ray geometry to change. The associated change in the phase path is given by

$$\begin{aligned}
\delta_m P &= \int_A^B \left(\delta_m \mathbf{p} \cdot \frac{d}{du} \mathbf{x} + \mathbf{p} \cdot \delta_m \frac{d}{du} \mathbf{x} \right) du \\
&= \int_A^B \left(\frac{1}{2} \mu_{m.p.}^2 + \frac{d}{du} [\mathbf{p} \cdot \delta_m \mathbf{x}] \right) du
\end{aligned} \quad (4)$$

where $\delta_m \mathbf{x}$ represents changes in a point on the ray. The result is written in a more general form than that required in the present case. Note that the first variation of the phase path is only affected by changes in ray geometry through the end point terms. However, in our case the ray end points are fixed and $\delta_m \mathbf{x} = 0$ at both end points.

Since the group path is related to the frequency derivative of the phase path, it is possible to write

$$P' = P + f \frac{\partial P}{\partial f} = P + \delta_f P \quad (5)$$

if the operator δ_f is defined by $\delta_f = \frac{\partial}{\partial f}$. Thus, the first order change in group path becomes

$$\delta_m P' = \delta_m P + \delta_m \delta_f P \quad (6)$$

In the present situation, it can be shown the general formula for $\delta_m \delta_f P$ reduces to

$$\delta_m \delta_f P = \int_A^B \frac{1}{2} \left(f \frac{\partial \mu_m^2 \delta m}{\partial f} + \frac{\partial \mu_m^2 \delta m}{\partial z} \delta_f z - \left(\frac{\delta_f p_x}{p_x} + f \frac{\partial \mu_{m.p.}^2}{\partial f} \frac{1}{\mu_{m.p.}^2} \right) \mu_m^2 \delta m \right) du \quad (7)$$

Evaluation of this expression requires a knowledge of $\delta_f z$ and $\delta_f p_x$. We return to this point later, but remark $\delta_f p_x$ is a constant for a given path and frequency, and that $\delta_f z$ is symmetric about the midpoint.

Notice that in the *particular case* of ionospheric structure which is *anti-symmetric* about the mid-point, $\delta_m P = 0$: there is no first order change in phase path. Since this result is identically true for all frequencies propagating over the given path, it is also clear that no first order change in group path. This can be confirmed from (7). Further, since the maximum frequency (or frequencies) capable of propagating over a given path are determined by the condition that

$$\frac{\partial P'}{\partial f} = \infty \quad (8)$$

it follows that the maximum frequency is also constant to the first order in this case.

Variation of the ray path

The variation of the ray path can be determined by solving the equations

$$\begin{aligned} \frac{d}{du} \delta_m p_x &= \frac{1}{2} \frac{\partial \mu_m^2 \delta m}{\partial x} & \frac{d}{du} \delta_m x &= 0 \\ \frac{d}{du} \delta_m p_y &= \frac{1}{2} \frac{\partial \mu_m^2 \delta m}{\partial y} & \frac{d}{du} \delta_m y &= \delta_m p_y \\ \frac{d}{du} \delta_m z &= \delta_m p_z - \frac{p_z \delta_m p_x}{p_x} \\ \delta_m p_z &= \left(\frac{\partial \mu_{m.p.}^2}{\partial z} \delta_m z + \mu_m^2 \delta m - 2 p_x \delta_m p_x \right) / 2 p_z \end{aligned} \quad (9)$$

where all derivatives are determined at points on the unperturbed (symmetrical) ray. The solution of this system requires some effort.

We are now in a position to discuss the equations determining $\delta_f z$ and $\delta_f p_x$. They take a form similar to (9), with f replacing m . However, there are no horizontal gradients involved in the frequency variation of the medium. Thus

$$\begin{aligned} \frac{d}{du} \delta_f p_x &= 0 & \frac{d}{du} \delta_f x &= 0 \\ \frac{d}{du} \delta_f p_y &= 0, \delta_f p_y = 0 & \frac{d}{du} \delta_f y &= 0 \\ \frac{d}{du} \delta_f z &= \delta_f p_z - \frac{p_z \delta_f p_x}{p_x} \\ \delta_f p_z &= \left(\frac{\partial \mu_{m.p.}^2}{\partial z} \delta_f z + \mu_f^2 f - 2 p_x \delta_f p_x \right) / 2 p_z \end{aligned} \quad (10)$$

Variation of initial and final ray directions

Frequently the actual path of the ray is not of so much interest as are the initial and final ray directions, for a ray between given end points. It is possible to determine these changes without actually solving the system (9). The result depends upon the second variation of the phase path, and the fact that the initial and final ray vectors can also be represented as variations of the phase path. Taking the ray vector at B for example, it is possible to write

$$\begin{aligned} \delta_m p_{xB} &= \frac{\sqrt{1-p_x^2}}{p_x} \delta_m p_{zB} \\ \delta_m p_{yB} &= \int_A^B \frac{1}{2} \left(\frac{\delta_n y}{\delta_n y_B} \frac{\partial \mu_m^2 \delta m}{\partial y} \right) du \\ \delta_m p_{zB} &= \int_A^B \frac{1}{2} \left(\frac{\delta_n z}{\delta_n z_B} \frac{\partial \mu_m^2 \delta m}{\partial z} - \frac{\delta_n p_x}{\delta_n z_B} \frac{\mu_m^2 \delta m}{p_x} \right) du \end{aligned} \quad (11)$$

where δn represents a ray variation which does not involve a variation of the medium. Further details of the derivation are given by [2]. From (11)

$$\begin{aligned}\frac{d}{du}\delta_n p_x &= 0 & \frac{d}{du}\delta_n x &= 0 \\ \frac{d}{du}\delta_n p_y &= 0 & \frac{d}{du}\delta_n y &= \delta_n p_y \\ \frac{d}{du}\delta_n z &= \delta_n p_z - \frac{p_z \delta_n p_x}{p_x}\end{aligned}\quad (12)$$

$$\delta_n p_z = \left(\frac{\partial \mu_{m.p.}^2}{\partial z} \delta_n z - 2 p_x \delta_n p_x \right) / 2 p_z$$

In (11), $\delta_n y$ which is involved in the transverse or out-of-plane variation is a solution for which $\delta_n x_A = \delta_n y_A = \delta_n z_A = 0$ and $\delta_n p_{xA} = \delta_n p_{zA} = 0$. It is given simply by

$$\frac{\delta_n y}{\delta_n y_B} = \frac{x - x_A}{x_B - x_A}$$

On the other hand $\delta_n z$, $\delta_n p_x$, $\delta_n p_z$ involved in the in-plane variation represent a solution for which $\delta_n x_A = \delta_n y_A = \delta_n z_A = 0$ and $\delta_n p_{yA} = 0$. They are not so easily evaluated.

The change or variation in zenith angle and bearing can be obtained from $\delta_m \mathbf{p}$. For example

$$\begin{aligned}\delta_m \theta_A &= \frac{\delta_m p_{xA}}{\sqrt{1 - p_x^2}} \\ \delta_m \phi_A &= \frac{\delta_m p_{yA}}{p_x}\end{aligned}$$

It is useful to obtain results applying specifically to the symmetric and anti-symmetric cases. The significance of the results is better understood with the aid of a sketch, Figures 1 and 2.

Symmetrical case

$$\begin{aligned}\delta_m p_{xA} &= \delta_m p_{xB} \\ \delta_m p_{yA} &= -\delta_m p_{yB} \\ \delta_m p_{zA} &= -\delta_m p_{zB}\end{aligned}\quad (13)$$

where

$$\begin{aligned}\delta_m p_{xB} &= \frac{\sqrt{1 - p_x^2}}{p_x} \delta_m p_{zB} \\ \delta_m p_{yB} &= \frac{1}{4} \int_A^B \frac{\partial \mu_m^2}{\partial y} \delta m \, du \\ \delta_m p_{zB} &= \int_A^B \frac{1}{2} \left(\left(\frac{\delta_n z}{\delta_n z_B} - \frac{1}{2} \right) \frac{\partial \mu_m^2}{\partial z} \delta m - \frac{\delta_n p_x}{\delta_n z_B} \frac{\mu_m^2 \delta m}{p_x} \right) du\end{aligned}\quad (14)$$

Anti-symmetrical case

$$\begin{aligned}\delta_m p_{xA} &= -\delta_m p_{xB} \\ \delta_m p_{yA} &= \delta_m p_{yB} \\ \delta_m p_{zA} &= \delta_m p_{zB}\end{aligned}\quad (15)$$

where

$$\begin{aligned} \delta_m P_{xB} &= \frac{1}{4} \int_A^B \frac{\partial \mu_m^2 \delta m}{\partial x} du \\ \delta_m P_{yB} &= \int_A^B \frac{1}{2} \left(\frac{x - \frac{1}{2}(x_B + x_A)}{x_B + x_A} \right) \frac{\partial \mu_m^2 \delta m}{\partial y} du \\ \delta_m P_{zB} &= \frac{P_x}{\sqrt{1 - p_x^2}} \delta_m P_{xB} \end{aligned} \quad (16)$$

Observe that symmetrical ionospheric structure gives rise to a symmetric ray, as would be expected. Notice that two cases are particularly simple to calculate, the variation in bearing in the symmetric case, and the change in zenith (or elevation) angle in the anti-symmetric case.

Conclusion

The results obtained in this paper provide considerable insight into the effect of symmetric and anti-symmetric horizontal structure on oblique radio propagation via the ionosphere. They may also be useful in the rapid path calculations required in some applications such as single-station location and over-the-horizon radar.

To the extent that the linear approximation is valid, they also provide an expansion theorem for ionospheric radio propagation. The effects of various horizontal structures can be considered separately, and the results then combined.

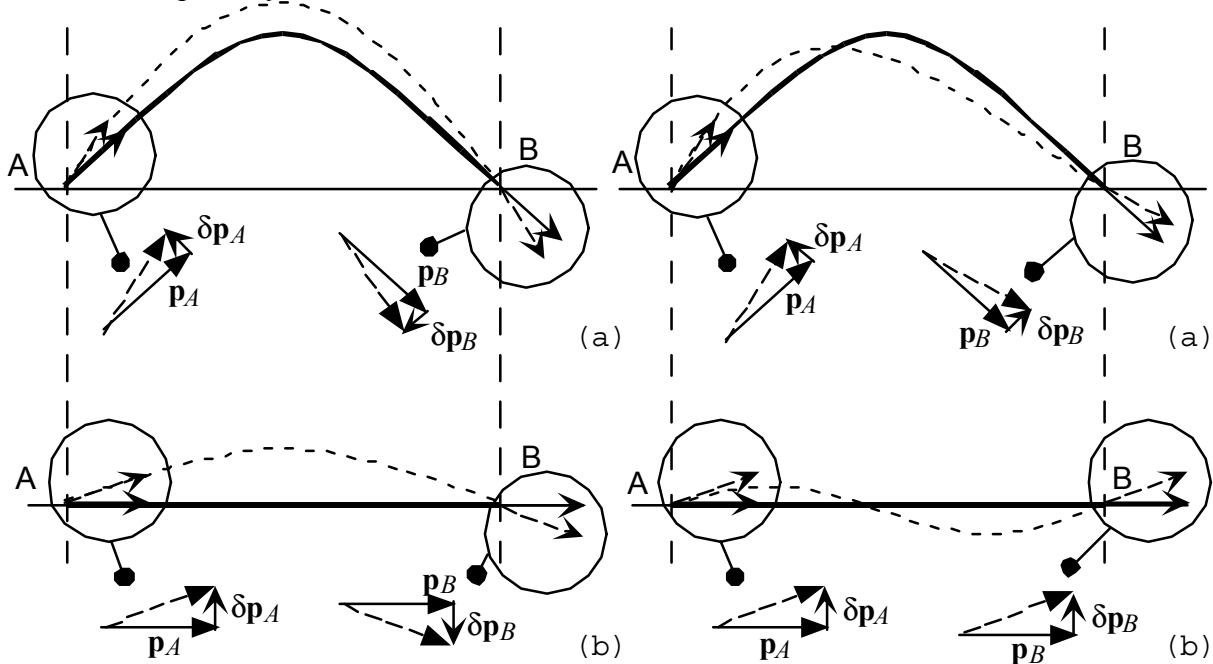


Figure1 : Ray geometry, symmetric horizontal ionospheric structure. (a) elevation (b) plan view

Figure2 : Ray geometry, anti-symmetric horizontal ionospheric structure. (a) elevation (b) plan view

References

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