A fast three-dimensional ray tracing formulation, with applications to HF communications and radar prediction

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Abstract. Fast two-dimensional ionospheric ray tracing has proved an effective tool for real-time applications in both over-the-horizon radar, and in HF communication systems. In some applications, three-dimensional ray tracing is desirable to obtain information about out-of-plane deviations. A two-dimensional ray tracing scheme has been extended to a full three-dimensions in order to handle such deviations. This scheme adds very little time penalty to the original. It has been used to investigate the out-of-plane effects of the terminator, and Travelling Ionospheric Disturbance phenomena (TID). The formulation uses cylindrical coordinates and neglects magneto-ionic effects.

1. Introduction

In a similar manner to long range communications, over-the-horizon radar, with a range of thousands of kilometres is possible due to the refraction of HF radio waves in the ionosphere. Whilst propagation in the ionosphere is a complex phenomena, it is usually sufficient to use geometric ray-tracing to model and analyse its behaviour.

It has been shown that for many applications, two-dimensional ray-tracing gives sufficient accuracy, with much better time performance than full three-dimensional ray-tracing [1]. This method has been used effectively in real over-the-horizon radar facilities, for simulating ionograms [2][3], and to perform real time calculations in the Coordinate Registration (CR) process [4]. CR is the conversion of radar coordinates (propagation time and beam azimuth) to geographic coordinates (latitude and longitude). In this two-dimensional case, propagation takes place in the plane of a great circle, containing the start and end points of the ray, and assumes that there is no out-of-plane deviation.

By extending this method to three-dimensional cylindrical coordinates, it is possible to obtain information about small out-of-plane deviations, due to horizontal gradients in the ionosphere, with little additional time penalty. A description of the derivation of three-dimensional ray-tracing equations is given in Section 2, with Section 3 going on to describe the ionospheric model used in this paper. Section 4 then describes an application of these equations to three-dimensional CR and coordinate prediction. Finally, Section 5 gives a numerical example of the application of these ray-tracing equations, by predicting out-of-plane deviations registered by a radar, due to a Travelling Ionospheric Disturbance (TID).

2. Derivation of Ray Tracing Equations

Equations that describe the propagation of radio waves can be derived using Fermat's principle of least time. Using the notation of the calculus of variations, and neglecting anisotropy (ie magneto-ionic effects) this principle can be expressed as:

$$\delta \int_A^B \mu \, ds = 0$$

Equation 1

where A and B specify the two end points of a ray, and μ is the refractive index of the ionosphere.

In cylindrical polar coordinates, $ds = \sqrt{dr^2 + r^2 d\theta^2 + dz^2}$, where r is the distance from the centre of the earth, θ is the angle in radians around the great circle, and z is the lateral coordinate. Thus, letting $r' = dr/d\theta$, $z' = dz/d\theta$, and $L = \mu\sqrt{r'^2 + r^2 + z'^2}$ the following two Lagrange equations can be derived from Equation 1:

$$\frac{d}{d\theta}\left(\frac{\partial L}{\partial r'}\right) - \frac{\partial L}{\partial r} = 0 \qquad \frac{d}{d\theta}\left(\frac{\partial L}{\partial z'}\right) - \frac{\partial L}{\partial z} = 0$$

Equation 2 **Equation 3**

Letting $Q = \mu r'\left(r^2 + r'^2 + z'^2\right)^{-\frac{1}{2}}$ and $W = \mu z'\left(r^2 + r'^2 + z'^2\right)^{-\frac{1}{2}}$, and using the fact that $dP' = \mu^{-1} ds$, the following system of equations can be derived from Equations 2 and 3 in terms of the group path, P':

$$\frac{dr}{dP'} = Q$$

$$\frac{dQ}{dP'} = \frac{1}{2}\frac{\partial \mu^2}{\partial r} + \frac{\mu^2 - Q^2 - W^2}{r}$$

$$\frac{d\theta}{dP'} = \frac{\sqrt{\mu^2 - Q^2 - W^2}}{r}$$

$$\frac{dW}{dP'} = \frac{1}{2}\frac{\partial \mu^2}{\partial z}$$

$$\frac{dz}{dP'} = W$$

Equations 4-8

These five equations are sufficient for specifying the coordinates of a ray, in terms of r, z and θ, at every point along its group path P', provided that initial conditions, and the refractive index of the ionosphere at that point are known.

In general, the refractive index is a function of r, z and θ, and can be expressed as

$$\mu(r,\theta,z) = \left(1 - \frac{\beta N(r,\theta,z)}{f^2}\right)^{\frac{1}{2}}$$

Equation 9

where $N(r,\theta,z)$ is the electron density (electrons per cubic centimetre), f is the wave frequency (MHz), and β=8.05x10^{-5} (ie a constant with units of $MHz^2.cm^3$) [5].

3. Ionospheric Model

The present work uses an ionospheric model based on three Chapman layers, E, F1 and F2 [3]. In this model, the electron density is a function of the height above the earth and is spherically symmetric. Therefore in a cylindrical coordinate system, the electron density is a function of r, but not of z or θ, and so $z' = W = 0$. In this case, Equations 7 and 8 can be discarded, and the three remaining equations reduce to the simpler two-dimensional equations given in [2]. Effects due to the earth's magnetic field are ignored. The full equation for the electron density, $N(r)$, is given in [3].

A lateral gradient in the ionosphere can easily be introduced to the model, by letting some of the ionospheric layer parameters depend on z, as in the numerical examples of Section 5. Thus, the electron density can be written as $N(r,z)$, and the derivatives of μ^2 required in Equations 4 to 8 can then be obtained from Equation 9, by squaring and differentiating:

$$\frac{\partial\mu(r,z)^2}{\partial r} = -\frac{\beta}{f^2}\frac{\partial N(r,z)}{\partial r}$$

Equations 10

$$\frac{\partial\mu(r,z)^2}{\partial z} = -\frac{\beta}{f^2}\frac{\partial N(r,z)}{\partial z}$$

Equation 11

If a ray originates at sea level, the initial conditions are that r is the radius of the earth, $Q = \sin\phi$, (ϕ being the initial elevation angle of a ray), and $W = \theta = z = 0$. Therefore, all the information required for solving Equations 4-8 numerically is known. The current work uses the Runge-Kutta Fehlberg (RKF) method [6] for solving this system of equations, and it has been found that this method gives a good combination of speed and accuracy.

4. Application to Coordinate Registration and Prediction

In an over-the-horizon radar facility, the data obtained directly from the radar is usually given in terms of the beam azimuth (the compass direction from which a received beam has arrived), and the time of flight of a pulse along a ray. It is straightforward to obtain the group path of an outward ray from the time of flight since $P'=0.5ct$, with c being the speed of light.

The azimuth and group path for a target are not particularly useful by themselves, and need to be converted into latitude and longitude. Assuming that there is no lateral deviation, the direction of the target is in the same direction as the beam azimuth. By solving the two-dimensional ray tracing equations, it is simple to approximate the actual ground range of a target that corresponds to a particular group path, and hence the targets latitude and longitude [2]. Note that it is assumed that a target is at ground level, or relatively close to it, and therefore a ray is always traced from its origin to where it hits ground level again.

If, however, a lateral electron density gradient is present, the beam azimuth will not give the actual bearing of the target. Although the two-dimensional method will still give a good approximation to the ground range, provided the lateral deviation is small, Equations 4-8 can be used to provide values for the actual bearing of the target, and more accurate values for the ground range.

For the event of a time varying lateral gradient in the ionosphere, an algorithm has been developed using these equations to perform the CR process in three dimensions, and hence find the real target bearing, ω, and ground range, D, given radar coordinates of P' and Ψ, and elevation angle, ϕ.

Conversely, given that the true path of a target is known in terms of its bearing, ω, and ground range, D, another algorithm can be used for predicting the beam azimuth, Ψ, and elevation angle, ϕ, required for tracking the target over a certain time period. This has been called the Tracking Prediction Algorithm. An example of the use of this algorithm is given in Section 5.

5. A Numerical Example

Consider the following scenario. A jet is flying in a northerly direction at 600km/h. Directly south of this jet is an over-the-horizon radar system. This radar starts tracking the jet at a distance of 1000km away, until it is about 1600km away. If the jet is tracked constantly for an hour, then the radar should show that the jet has travelled about 600km, in a straight line with a constant velocity. If, however, there is a non-zero lateral gradient, the radar facility may show the bearing of the fighter as being some degrees east or west of north.

We introduce a lateral gradient, in the form of a TID in the F2 layer, with velocity v (km/h), and wavelength l (km). A TID in a direction lateral to a propagating ray will modify the plasma frequency [7] of the F2 layer, foF2, in a manner that can be modelled as:

$$foF2(z,t) = foF2_0\left[1+\delta \sin\left(\frac{z-vt}{l}\right)\right]$$

Equation 12

The following plots were obtained by running the tracking prediction algorithm for the above scenario, with samples every 18 seconds, and $foF2_0$ = 10 MHz, l = 40km, δ = 0.1 and v = 100 m.s^{-1}. Note that this value of δ gives a variation of ±10% in foF2, ie a *strong* TID [7][8], whilst a TID with velocity 100 m.s^{-1} is classified as medium-scale [9].

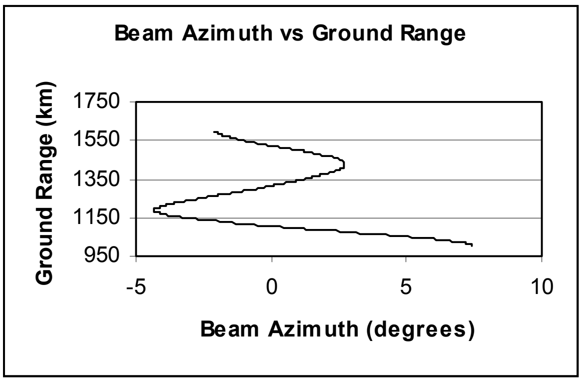

Figure 1

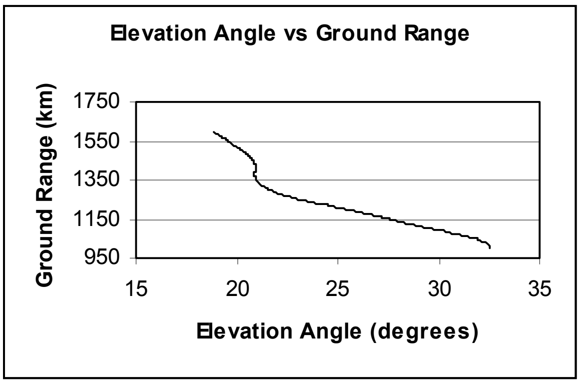

Figure 2

Figure 1 shows the predicted beam azimuth required for tracking the target over time, plotted against ground range. Figure 2 shows the predicted elevation angle required, plotted against ground range. Note that ground range in these plots can easily be converted into time, since the jet is moving northwards at a velocity of 600km/h.

The higher lateral deviation shown at a closer range is due to the larger elevation angle required. A larger elevation angle means that the ray travels higher in the ionosphere, and is more affected by the TID than a ray at a lower elevation angle. To illustrate this, Figures 3 and 4 show the trajectory taken by a single ray, at time t = 0, at elevation angles of 20°, and 30° and a frequency of 14 MHz.

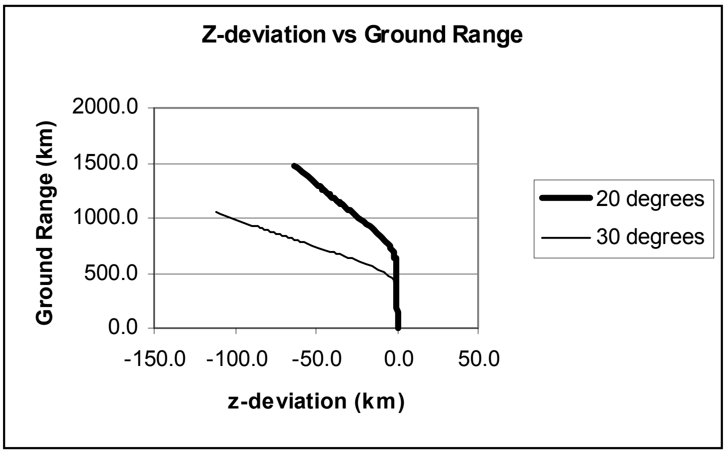

Figure 3

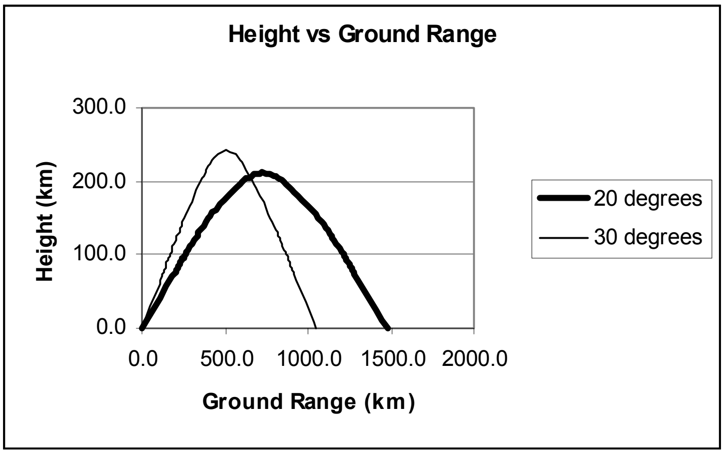

Figure 4

6. Time comparison for the extended ray-tracing equations

The addition of the third dimension to the ray-tracing equations necessitates some extra CPU time when calculating ray paths. For a comparison in the run time required for ray-tracing with the extended equations with the original two-dimensional equations, the RKF method was used to find trajectories of rays at fifty different elevations, for 100 different times. In the three-dimensional case, the lateral gradient resulting from Equation 12 meant each time gave a different solution. Obviously, in this case, the two-dimensional case had the same solution for each time.

On a Pentium III 550MHz processor, under Windows NT, the three-dimensional equations took about 9.7 seconds to trace the 5000 rays. The two-dimensional version took 8.2 seconds. This equates to only an 18% increase in time for the calculation of out-of-plane deviations. This increase is still much less than that incurred by use of another more traditional three-dimensional ray-tracing technique [1].

7. Conclusion

This paper has described how a two-dimensional ray-tracing algorithm has been extended to three-dimensions, to allow for the calculation of out-of plane deviations. This three-dimensional system of equations adds little time penalty to ray-tracing applications, and therefore is suitable for use in real-time radar applications, as an example problem has illustrated. In current work, these three-dimensional equations have been used with more general ionospheres, to show the out-of-plane effects of the terminator. Further work on this formulation could investigate the incorporation of magneto-ionic effects.

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