

Microstrip Circuit Analysis using the New Closed-Form Green's Functions

Yuehe Ge and Karu P. Esselle

Abstract—An efficient full-wave spatial domain numerical method is presented for the analysis of multilayer microstrip structures using new closed-form Green's functions in conjunction with the method of moment (MoM). This method solves the mixed potential integral equation (MPIE) for the surface current density on the microstrip. Using the new closed-form Green's functions and selecting roof-top functions as both basis and testing functions, one can derive closed-form expressions for the matrix elements involved in the MoM and improve the computational efficiency significantly without compromising the precision. The results from the new method agree well with the results from alternative methods.

Keywords—Sommerfeld Integrals; Green's functions; Method of Moments

I. INTRODUCTION

ONE of the most useful numerical techniques in the rigorous analysis of multilayer microstrip structures is the method of moments, which can be applied in either the spectral [1] or spatial domain [2], [3]. Although the spatial-domain MoM is preferred because it is relatively efficient in terms of the computational time, it is still time consuming because of the slow convergence and the oscillatory nature of the Sommerfeld integrals involved in the spatial-domain Green's functions. Since the closed-form Green's functions have been proposed to accelerate the calculation of the spatial-domain Green's functions, the computation of the MoM matrix elements has been sped up a great deal [4], [5]. In this approach, the Green's functions are approximated by a sum of complex exponentials. Then one can evaluate the MoM matrix elements analytically [6] and hence improve computational efficiency. However, this approach requires a Taylor-series approximation, which could lead to some loss of precision.

In this paper, a Galerkin's MoM has been developed for multilayer microstrip structures by employing a new set of closed-form Green's functions proposed by the authors recently [7], [8]. The method is used to solve the MPIE for microstrip structures. Because of the special form of our Green's functions, the matrix elements can be obtained analytically without any other approximations. Hence, an improved computational efficiency and better precision can be obtained.

II. THEORY AND NUMERICAL METHOD

A. MPIE Formulation

Consider a planar multi-layer microstrip structure consisting of several infinite substrates and conductors, all par-

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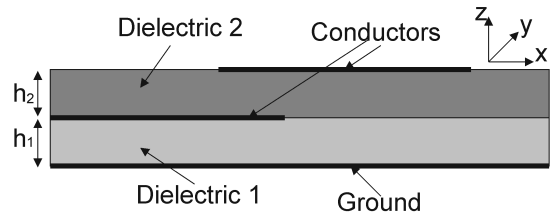


Fig. 1. A multilayer microstrip structure

allel to the x-y plane, as shown in Fig. 1. Each layer can have different electric and magnetic properties (ϵ_i, μ_i) and thicknesses (h_i). The conductors can have arbitrary two-dimensional shapes and can be located at any interface or within any layer. The mixed potential integral equation (MPIE) for a planar layered microstrip structure is given by :

$$\begin{aligned} \hat{e}_z \times \vec{E}_j^e(\vec{r}) &= \hat{e}_z \times \left[j\omega \int_{S_j} \vec{J}_{S_j}(\vec{r}') \bullet \vec{G}_j^A(\vec{r}|\vec{r}') ds' \right. \\ &+ \nabla \int_{S_j} q_{S_j}(\vec{r}') G_j^V(\vec{r}|\vec{r}') ds' \\ &\left. + Z_s \vec{J}_{S_j}(\vec{r}') \right] \end{aligned} \quad (1)$$

where \vec{r} is the position vector defined with respect to a global coordinate origin, \hat{e}_z is the outer normal vector on the surface S_j , \vec{J}_{S_j} is the electric surface current density on S_j , q_{S_j} is the charge density, \vec{G}_j^A and G_j^V are the Green's functions for the vector and scalar potentials, respectively, and Z_s is the surface impedance of the conductor.

Then we apply the Galerkin's MoM to solve the MPIE for the electric current density in the conductors. After selecting the basis and testing functions, we expand the current densities and then test them. Solving the resulting matrix equation, we find the current density distributions on the microstrip.

B. The New Closed-Form Green's Functions

The spatial-domain Green's functions for multilayer structures can be expressed by a shorthand form

$$S_0[f] = \frac{1}{4\pi} \int_C H_0^{(2)}(k_\rho \rho) k_\rho f(k_\rho) dk_\rho \quad (2)$$

where $H_0^{(2)}$ is the Hankel function of the second kind and order zero. The ρ is the radial distance in the x-y plane

between the field points and source points, k_ρ is the radial propagation constant in the x-y plane and C is the integration path. The function $f(k_\rho)$ is the spectral-domain Green's function, which can be obtained analytically for a multilayer medium.

These integrals, also called the Sommerfeld integral, cannot be evaluated analytically. However, we obtained closed-form expressions for them by using the method described in [7], [8], which have the form of

$$S_0[f] = \sum_{n=1}^N \frac{a_n b_n}{(b_n^2 + \rho^2)^{\frac{3}{2}}} \quad (3)$$

where $\rho^2 = x^2 + y^2$, a_n and b_n are two complex constants.

C. Closed-Form Expressions for MoM Matrix Elements

With a proper selection of basis and testing functions and using the new closed-form Green's functions, it is possible to completely eliminate numerical integration [9] in the calculation of matrix elements so that the efficiency is improved. In this paper, we use the roof-top function as both basis and testing functions. One of the typical matrix elements is

$$\langle W_{xm}, E(\bar{G}^A, P_{x'n}) \rangle = \iint_{S_w} W_{xm}(x, y) \cdot \{ \iint_{S_p} \bar{G}_{xx}^A(x-x', y-y') P_{x'n}(x', y') dx' dy' \} dx dy \quad (4)$$

where $P_{x'n}$, W_{xm} denote the basis and testing functions, respectively, S_w and S_p denote the domains of the testing and basis functions, respectively, and \bar{G}_{xx}^A denotes the Green's function related to the vector potentials and has the form of (3). By performing the variable substitution and changing the order of the integration, the inner product has the form of

$$\iint \bar{G}_{xx}^A(u, v) \{ \iint W_{xm}(x, y) P_{x'n}(x-u, y-v) dx dy \} du dv \quad (5)$$

The double integral between the two braces of (5) can be evaluated analytically. we have

$$\begin{aligned} & \iint W_{xm}(x, y) P_{x'n}(x-u, y-v) dx dy \\ &= k_1 u^3 v + k_2 u^2 v + k_3 uv + k_4 v + k_5 u^3 \\ &+ k_6 u^2 + k_7 u + k_8 \end{aligned} \quad (6)$$

where $k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8$ are constants determined by known parameters relative to the weighting and basis functions.

Substituting (3) and (6) into (4), (4) becomes the form below

$$\sum_{n=1}^N a_n b_n \iint \frac{u^i v^j}{(u^2 + v^2 + b_n^2)^{\frac{3}{2}}} du dv, \quad 0 \leq i \leq 3, \quad 0 \leq j \leq 1 \quad (7)$$

The double integrals in (7) are analytically integrable [6]. The other matrix elements can be evaluated with the same procedures.

III. RESULTS

A. Accuracy of the New Closed-Form Green's Functions

Consider a microstrip line with following parameters: the dielectric constant of the substrate $\epsilon_r = 10.2$; the thickness of the substrate $h = 0.64\text{mm}$; and the frequencies of operation are 6.88 GHz and 11.6 GHz, respectively. The Green's functions of the vector potential due to a horizontal electric dipole (HED) at the air-substrate interface are calculated using our closed-form and direct numerical integration, respectively. Fig. 2 shows the results and an excellent agreement is found.

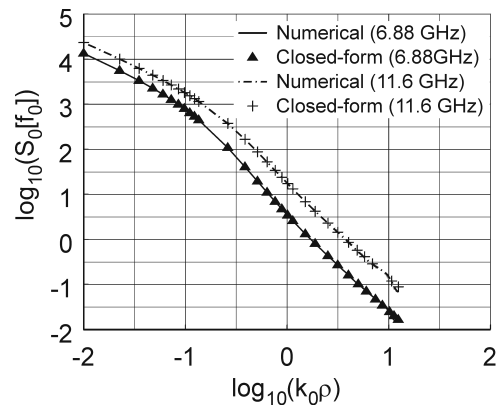


Fig. 2. The magnitude of the Green's functions

B. A Microstrip Transmission Line

We calculated the current distribution on a lossless, open-ended microstrip line, which has the following parameters: the dielectric constant of the substrate = 4; the thickness of the substrate = 0.203 mm; the width of the microstrip line = 0.81 mm; the frequency of operation = 1 GHz; the length of the microstrip line = 11 cm; the current source is located 1 cm away from the left edge, and has a magnitude of 2 A. Fig. 3 shows the magnitude of the current distribution along the microstrip line computed using the new MoM and the transmission line (TL) theory. It can be seen that the two results compare very well.

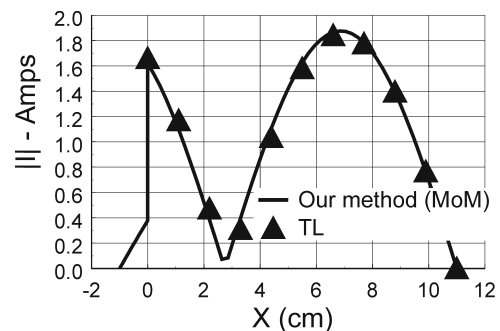


Fig. 3. Current distribution on a microstrip transmission line

We also used two previous MoM techniques to analyse the same example and compared the computational times with that of our method. The details has been published in [9]. The results showed that our method is about three

times faster than the next best method, which requires some numerical integration.

C. A Two-Gap Microstrip Filter

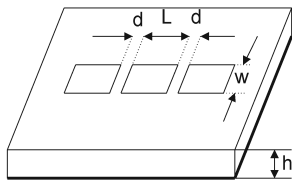


Fig. 4. Geometry of the microstrip filter

Next, consider a two-gap microstrip filter shown in Fig. 4. The substrate of the circuit has a thickness of $h=0.7874$ mm and dielectric constant of 2.33. The transmission line width $w=2.3$ mm, $d=0.3$ mm and $L=10.0$ mm. For comparison, we also analyzed the same filter using Ansoft Ensemble (SV) software. Fig. 5 shows the scattering parameters $|S_{11}|$ and $|S_{21}|$ from the two methods and a good agreement can be seen.

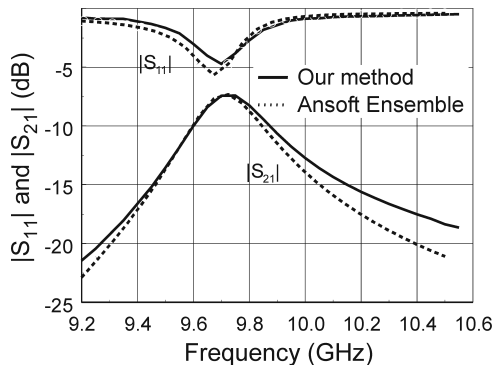


Fig. 5. Magnitude of scattering coefficients S_{11} and S_{21}

D. A Microstrip-Fed Slot Antenna

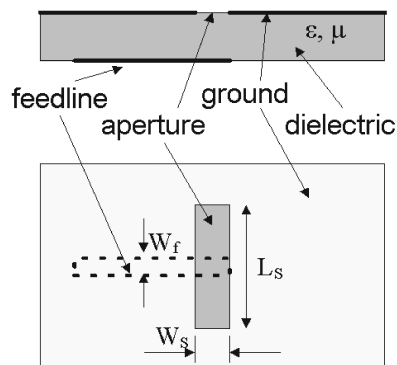


Fig. 6. Geometry of the slot antenna

Finally, to illustrate the versatility of our method, we analysed a slot antenna fed by a microstrip line, shown in Fig. 6. The slot is cut on the ground plane. The feedline is

on the other side of the substrate. The antenna's parameters are: $\epsilon = 2.5$, $d = 0.79$ mm, $L_s = 8$ mm, $W_s = 2.5$ mm and $W_f = 0.25$ mm. Fig. 7 shows the results of $|S_{11}|$ from our method and Ansoft Ensemble (SV) software for this antenna. Again, a good agreement can be observed.

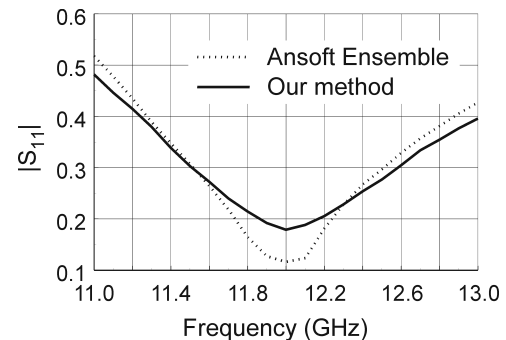


Fig. 7. Magnitude of scattering coefficients S_{11} for the slot-antenna

IV. CONCLUSIONS

In this paper, a full-wave spatial-domain MoM technique together with new closed-form Green's functions have been employed for the analysis of several planar microstrip structures. In all applications, all MoM matrix elements were evaluated without any numerical integration. This approach led to significantly better computational efficiency and good accuracy.

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