A Simulation Platform for Single and Multi-Channel Transmission Systems in Frequency Domain Using Volterra Series Transfer Functions

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Abstract **– We present a novel simulation model for optical communication systems in which single or multi- optical channels can be transmitted without divergence due to optical nonlinear effects. The Volterra Series Transfer Function (VSTF), represented by functional series, is used to obtain analytic solution of the Nonlinear Schrondinger (NLS) wave equation which represents the evolution of optical data pulses along the transmission fibre. The radius of convergence of the VSTF model is critical for the model and is described. Simulated results are compared with those using the well-known split-step Fourier method confirms the model efficiency and accuracy for optical pulse transmission.**

I. INTRODUCTION

Fiber-optic communication systems offer transmission capacity reaching tera-b/s over long haul distance have been continuously deployed across the world, inter-city and metropolitans. Since the advent of erbium-doped fiber amplifiers (EDFAs), dispersion management techniques, and various schemes for countering the loss and nonlinearity in the fibre, the transmission bit rate has increased exponentially from a few tens of Mb/s second to several tera-Hz/s [7]. With the increase in the signal bit rate/bandwidth, the transmission distance and especially the concurrent optical multiplexing of several information carrier channels, the system design becomes more and more complex. In system simulation the usual method, the split-step Fourier (SSF) method is employed. This is adequate for single or few channels propagation due to extensive number of steps of FFT and inverse-FFT. It is thus essential to develop an efficient model for simulating the fiber-optic communication systems.

The main difficulty in modeling the system lies in the representation of the nonlinear effect. Optical nonlinearities have long been recognized as key limitations in fibre optic transmission systems including self-phase modulation (SPM), cross-phase modulation (CPM), stimulated Raman scattering (SRS), stimulated Brillouin scattering (SRS) and four-wave mixing (FWM) that cause signal distortion and cross-talk.

The widely adopted SSF method is originally designed to incorporate the contribution of nonlinear effects in the temporal domain with the pulse evolution due to linear dispersive effects in the frequency domain. The split-step method is efficient for single channel system, i.e. only one optical carrier is modulated and transmitted over the optical medium. However, it is a recursive method and hence can be very inefficient for multi-channel transmission.

Volterra series transfer function (VSTF) represents a pseudo-analytic form of a nonlinear wave propagation equation [1] in which first and higher order series (called as kernels in frequency domain), form an approximate solutions. It has been employed in many nonlinear engineering problems for its intuitive representation of nonlinear contribution. Unlike other numerical methods such as those using SSF, the VSTF is non-recursive. This method has first been used for modeling of optical communication systems in 1997 [2-4]. However the convergence of such series representation has not been investigated and this is very critical for nonlinear pulse propagation, especially when the total average power of the wavelength multiplexed channels exceeds that allowable for single mode optical fibre, approximately about +5 dBm.

In order to determine whether the model is appropriate for modeling DWDM optical transmission systems, it is necessary to examine the convergence of the kernels of the VSTF corresponding to the evolution of the pulse peak power and the transmission distance. This paper thus develops a novel VSTF model and examines its convergence properties for propagation of optical data pulses for systems operating in the 40 Gb/s region. These results are compared with those obtained using SSF method and are shown to be consistent and allow us to determine accurately the upper limit of the optical power to be launched at the input of the transmission medium so that the pulses would not be diverged.

The results for non-dispersion shifted fibre (NDSF) transmission systems will be presented together with an examination of the validity of the VSTF model.

This paper is organized as follows. In the next section a brief description of the pulse propagation in the optical medium is given. Section 3 gives the detailed description of the VSTF representation of the pulse propagation. In section 4 the derivation of the radius of convergence criteria is presented. In section 5 a comparison between VSTF and SSF methods is given.

II. FIBER NONLINEARITIES – NONLINEAR SCHRODINGER (NLS) EQUATION

The generalized nonlinear Schrödinger wave equation that represents the propagation of the envelop of an optical pulses under the influence of both linear dispersion effect and nonlinear phase evolution can be given by

$$\frac{\partial A}{\partial z}+\frac{\alpha_0}{2}A+\beta_1\frac{\partial A}{\partial t}+j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}+\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3}$$
$$=j\gamma|A|^2A-a_1\frac{\partial\left(|A|^2A\right)}{\partial t}+a_2\frac{\partial A}{\partial t}|A|^2+a_3\frac{\partial|A|^2}{\partial t}A \qquad (1)$$
$$+\,jQ_RA\int_{-\infty}^{\infty}s_r(t-t_1)|A(t_1,z)|^2dt_1$$

where $A = A(t,z)$ represents the slowly-varying complex envelope of the optical pulses; α is the linear attenuation factor; β_1 is the group velocity or purely the propagation delay; β_2 is the linear dispersion factor, β_3 is the second order dispersion factor (i.e. the fibre dispersion slope with respect to the optical channel wavelength), γ is the SPM factor; a_1, a_2 and a_3 is the higher order nonlinear coefficients (related to higher order nonlinear effects such as self-steepening, shock formation, and Raman scattering); and finally Q_R is the contribution factor due to the Raman gain effect of the fibre. The higher order nonlinear effects are not included in this work and will be investigated in systems where Raman distributed amplifiers are included.

The NLS equation cannot be solved analytically due to its nonlinear nature. Normally, numerical methods such as split-step Fourier (SSF) or Runga-Kutta methods are used to obtain its solution. They are efficient for the case of single channel transmission but would require extensive computing time and resources to reach final solutions. We note that for Tb/s DWDM systems there may be 256 optical multiplexed channels concurrently propagate in the optical medium. Further these numerical techniques do not provide intuitive insight of the cross channel inter-modulation effects. These problems would be overcome by the VSTF model, which is described in the next section.

III. The VSTF Model

The Volterra series model provides a summation of a number of series which can approximate the wave equation in the frequency domain. The linear dispersion effect is represented via the first order kernel, and the nonlinear contribution due SPM and cross phase modulation or Raman scattering effects via the third order and higher order kernels. The transfer function of an optical pulse envelope between the input and output of a transmission fibre in the frequency domain, i.e. $X(\omega)$ and $Y(\omega)$ respectively, can be given by

$$Y(\omega) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} H_n(\omega_1, \ldots, \omega_{n-1}, \omega - \omega_1 - \ldots - \omega_{n-1}, z) \times X(\omega_1) \ldots X(\omega_{n-1}) X(\omega - \omega_1 - \ldots - \omega_{n-1}) d\omega_1 \ldots d\omega_{n-1} \quad (2)$$

where $H_n(\omega_1, \ldots, \omega_n)$ is the nth-order frequency domain Volterra kernel [1]. Equation (2) can in fact be represented as a summation of first and higher order responses as can be seen in the next section (equation (6)). The NLSE wave propagation inside a single-mode fiber can be simplified by eliminating the non-interest and negligible nonlinear terms as

$$\frac{\partial A}{\partial z} = -\frac{\alpha_0}{2} A - \beta_1 \frac{\partial A}{\partial t} - j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} + j\gamma |A|^2 A \quad (3)$$

Therefore the first order and 3rd order kernels can be obtained as

$$H_1(\omega, z) = e^{G_1(\omega) z} = e^{\left(-\frac{\alpha_0}{2} + j\beta_1\omega + j\frac{\beta_2}{2}\omega^2 - j\frac{\beta_3}{6}\omega^3\right)z} \quad (4)$$

$$H_3(\omega_1, \omega_2, \omega_3, z) = G_3(\omega_1, \omega_2, \omega_3) \times \frac{e^{\left(G_1(\omega_1) + G_1^*(\omega_2) + G_1(\omega_3)\right)z} - e^{G_1(\omega_1 - \omega_2 + \omega_3)z}}{G_1(\omega_1) + G_1^*(\omega_2) + G_1(\omega_3) - G_1(\omega_1 - \omega_2 + \omega_3)} \quad (5)$$

Other higher order terms can be included if more accuracy is required. As described the Volterra series transfer function of an optical transmission system takes the form of a power series, and naturally its convergence is very critical. This is investigated in the next section and is the central development of this work. The subscripts 1,2 and 3 of the ω terms are dummy variables that should be scanned across the frequency spectra covering all the wavelength channels. In DWDM systems this spectrum can be extended from the S-band to the L-bands, i.e. 1485 nm to 1625 nm.

IV. Radius of Convergence

In order to ensure that VSTF is converging during the propagation of the pulses, there are a number of tests which are the sandwich criteria, ratio testing etc., can be used. The ratio test is chosen for the VSTF as it is more efficient and accurate. Eq. (2) can be re-written as a summation of all order terms, that is an infinite Volterra series in the frequency domain as [8]

$$\sum_{n=1}^{\infty} Y_n(\omega_1, \ldots, \omega_n) = \sum_{n=1}^{\infty} H_n(\omega_1, \ldots, \omega_n, z) X(\omega_1) \ldots X(\omega_n) \le \sum_{n=1}^{\infty} \left| H_n(\omega_1, \ldots, \omega_n, z) \right| \left| X_{\max} \right|^n \quad (6)$$

where $X_{\max} = \max\{X(\omega)\}$ the maximum allowable magnitude of the optical pulse launched at the input; $Y_n(\omega_1, \ldots, \omega_n)$ is the output from n^{th} order kernel or the frequency response of the system corresponding to appropriate disturbance, i.e. the linear dispersion and nonlinear effects. The equation can thus be written in the inequality form. The output response terms Y_1, Y_2, …, Y_n contain different dimensional frequency terms following the order of the kernels. The higher order kernels can be converted into its one-dimensional equivalent with a technique called dimensional contraction, that is to perform a multi-dimensional convolution across all the variables. Thus we can rewrite the LHS of (6) and then equates it with the RHS as:

$$\begin{aligned} Y(\omega) &= Y_1(\omega) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Y_3(\omega_1, \omega_2, \omega - \omega_1 - \omega_2) d\omega_1 d\omega_2 + \ldots \\ &= H_1(\omega, z) X(\omega) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_3(\omega_1, \omega_2, \omega - \omega_1 - \omega_2, z) \\ &\quad \times X(\omega_1) X(\omega_2) X(\omega - \omega_1 - \omega_2) d\omega_1 d\omega_2 + \ldots \end{aligned} \quad (7)$$

The upper bounds as stated in (6) of input power, the maximum allowable total optical peak power, for the kernels of the VSTF can be derived using triangular inequality and by algebraic manipulation:

$$X_{\max} = \inf\left\{\left(\frac{\left|H_1(\omega,z)\right|}{\left|\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} H_n(\omega_1,\ldots,\omega-\omega_1-\ldots-\omega_{n-1},z)\,d\omega_1\ldots d\omega_{n-1}\right|}\right)^{\frac{1}{n-1}}\right\} \quad (8)$$

This maximum peak power can be evaluated as a function of the transmission distance as shown in Fig. 1. It is noted that the distance is used a whole distance and no partition of the distance into differential intervals as usually conducted in other numerical techniques. This is the strength of the VSTF technique. The fibre properties used in the model calculations are: dispersion factor +15 ps/nm/km (standard value for non-dispersion shifted fibres NDSF), attenuation = 0.15 dB/km, dispersion slope = 0 ps2/nm/km, nonlinear coefficient n_2 = 2.31×10^{-20} m^2/W.

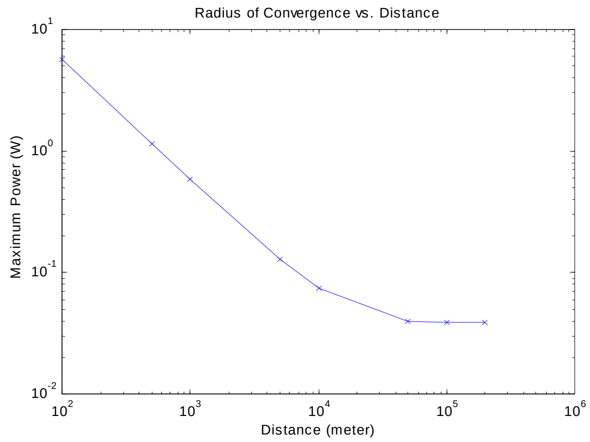


Figure 1. Maximum peak pulse power to be launched at the fibre input versus fibre transmission distance.

The curve in Figure 1 indicates the boundary for the convergence of the VSTF model. We observe that the longer the transmission distance the lower the maximum allowable peak power is. This is due to the competition between the linear dispersion represented by $H_1(\omega,z)$ kernel and the nonlinear phase, hence nonlinear dispersion accumulated along the transmission distance as contributed by $H_3(\omega_1,\omega_2,\omega_3,z)$. Unless there is a finite difference between these two dispersion effects the total response or the series can converge. The maximum peak power is saturated for distance reaching about 100 km because at this limit there is balance between the linear and nonlinear contribution.

V. PULSE PROPAGATION BY VSTF METHOD AND SSF METHOD

Input Gaussian pulses of different peak power are propagated through NDSF fibers of different length. The pulse evolution has been obtained using both the VSTF and the SSF methods. The difference between these two methods are compared using the criterion:

$$Deviation\ in\ \% = \frac{\int_{-\infty}^{\infty}\left|Y_{VSTF}(\omega)-Y_{SSF}(\omega)\right|^2 d\omega}{\int_{-\infty}^{\infty}\left|Y_{SSF}(\omega)\right|^2 d\omega} \quad (9)$$

where $Y_{VSTF}(\omega)$ and $Y_{SSF}(\omega)$ are the pulse spectrum at the output obtained from the VSTF method and the SSF method respectively. Fig. 2 and Fig. 3 show the deviation of the pulses after propagating over distances of 50 km and 100 km respectively. They are agreeable for pulse peak power not higher than 30 mW. However there is a large discrepancy when it is higher than this limit. In practice this convergence limit is within total input allowable power for return-to-zero carrier suppressed optical communication systems[8].

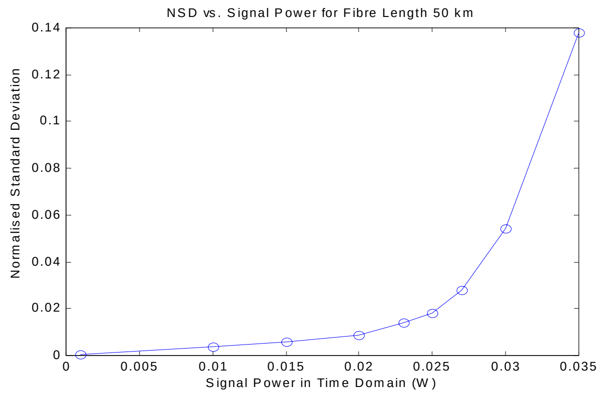


Figure 2. Deviation vs. Input Power for propagation distance of 50 Km.

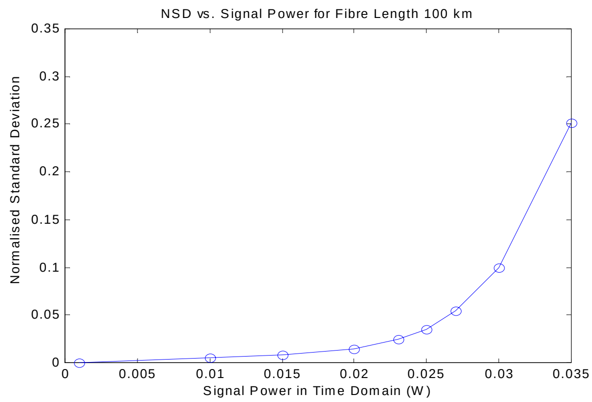
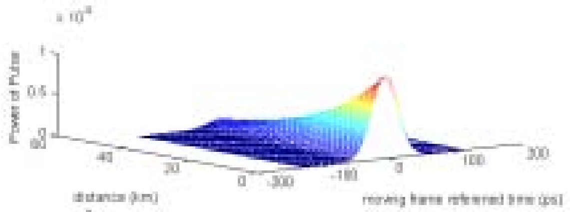


Figure 3. Deviation vs. Input Power for propagation distance of 100 Km.

Fig. 4 and Fig. 5 shows the pulse evolution for transmission distance of 50 km and 150 km respectively.



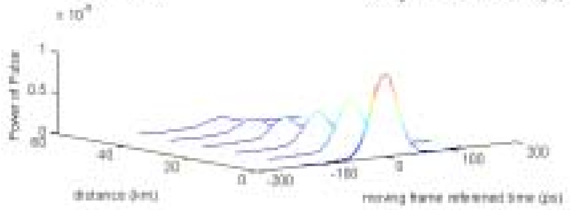


Figure 4. Simulation result obtained from SSF (upper curves) and VSTF (lower curves) methods respectively for 50 km of transmission distance and 1.0 mW of input peak power.

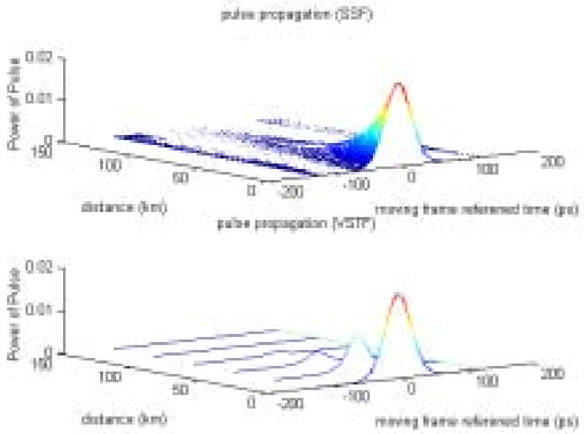


Figure 5. Simulation result obtained from SSF and VSTF methods respectively for 150 km of transmission distance and 10 mW of input peak power.

VI. Conclusion

The Volterra series transfer function has been demonstrated as a model for study the optical pulse propagation in single-mode fibre optical transmission systems. Both linear and nonlinear effects can practically be approximated by higher order Volterra kernels, possibly up to 5^{th} order. The competition between the linear and nonlinear dispersion effects has been observed via the radius of convergence of the series, i.e the contribution of the first and third order kernels of the series. This paper also provides a conservative criterion for determination of the upper bounds of the input pulse peak power under which the VSTF model remains convergent. It is consistent with average optical power normally used in practical transmission systems. The relationship between radius of convergence for input power and the fiber length is also described.

Although the multi-channel (DWDM) optical systems have not been investigated in details, the upper limit would allow us to determine the total average power of all optical channels for convergent transmission. Simulation results for these systems will be presented in a future article. The VSTF model would offer much more efficiency in modeling these multi-channel systems as demonstrated in this work. Furthermore as the VSTF is working in the frequency domain it allows us to synthesize the transfer functions of other optical sub-systems such as dispersion compensators, equalizers, filters etc.

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