Impact on Near-field Horn Gain from the Phase Curvature Variation in the Horn Aperture Field

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ABSTRACT – The difference in the theoretical near-field gain correction when using slant or axial length as the radius of curvature of the phase front over the pyramidal horn aperture plane is presented. The difference is an indication of the contribution to the measurement uncertainty, and is discussed for different operating frequencies, different horn sizes and different horn-to-horn separation distances. As an example, the analysis is applied to 10 GHz standard horn gain measurement data.

1. INTRODUCTION

According to the well-known induction theorem [1], the electromagnetic field generated by a horn antenna can be calculated by integrating the field distribution over the horn aperture. Therefore, obtaining a precise expression of aperture field distribution in both magnitude and phase is important.

For a pyramidal horn, linearly polarized in the y-direction, the field over the aperture (mouth) of the transmitting horn may be assumed to be similar to that of the dominant TE_{10} mode of the rectangular feed waveguide (throat), except that a phase term is included to account for the phase variation that the field exhibits across the aperture. The tangential component of E-field in the aperture (x, y) is taken as

$$E_1(x,y) = E_1 \cos(\frac{\pi x}{a}) \exp\{-jk[\delta_x(x) + \delta_y(y)]\} , \qquad (1)$$

where $\delta_x(x)$ and $\delta_y(y)$ describe the phase variation at *(x,y)*, E_1 is a constant, and a is the aperture dimension in the x-direction.

In general, the phase variation across the aperture is a complicated function, and for practical computation purposes, must be approximated. The phase term must primarily account for the difference in path length that the wave travels from the throat to different points in the horn aperture as shown in Fig. 1. Referring to the E-plane (yz-plane) view of horn system in Fig. 1 and using a binomial expansion, the phase variation in the y-direction is approximated by [1]

$$\begin{aligned} \delta_y(y) &= \sqrt{l_{ea}^{\ 2} + y^2} - l_{ea} \\ &\approx l_{ea}(1 + \frac{y^2}{2l_{ea}^{\ 2}}) - l_{ea} \\ &\approx \frac{y^2}{2l_{ea}} \end{aligned} \qquad (2a)$$

or may be approximated by [2, 3]

$$\begin{aligned}\delta_y(y) &= \sqrt{l_{ea}{}^2 + y^2} - l_{ea} \\ &= \sqrt{l_{es}{}^2 - (b/2)^2 + y^2} - \sqrt{l_{es}{}^2 - (b/2)^2} \\ &\approx l_{es}[1 + \frac{y^2 - (b/2)^2}{2l_{es}{}^2}] - l_{es}[1 - \frac{(b/2)^2}{2l_{es}{}^2}] \\ &\approx \frac{y^2}{2l_{es}}\end{aligned} \tag{3a}$$

where l_{es} and l_{ea} are the E-plane slant and axial lengths, respectively. l_{es} and l_{ea} can also be considered to be the maximum and minimum radii of curvature of the phase front over the horn aperture.

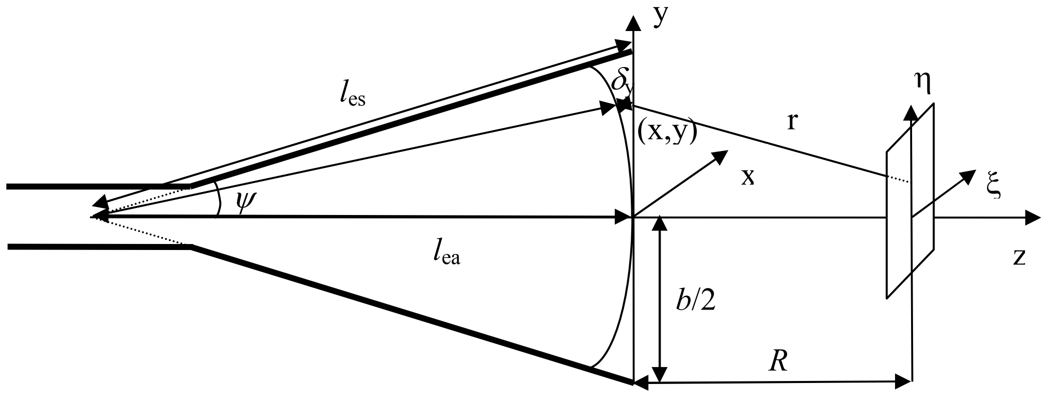


Fig. 1 E-plane view of transmitting pyramidal horn and coordinate system

Similarly, $\delta_x(x)$ may be approximated by

$$\delta_x(x) \approx \frac{x^2}{2l_{ha}} \tag{2b}$$

or

$$\delta_x(x) \approx \frac{x^2}{2l_{hs}} \tag{3b}$$

where l_{hs} and l_{ha} are the H-plane slant and axial lengths, respectively. Both expressions (2) and (3) are approximations, which introduce uncertainty in the theoretical horn gain.

A short discussion of the difference caused by using slant length or axial length in the near-field correction of horn gain was given by Hunter and Morgan [3], who suggested choosing intermediate values $l_e = \left[l_{ea}{}^2 + (0.3b)^2\right]^{1/2}$, $l_h = \left[l_{ha}{}^2 + (0.3a)^2\right]^{1/2}$.

In this paper, we will examine how the difference between these phase approximations affects the theoretical near-field correction. Because other uncertainty sources, such as reflections in the chamber, impedance mismatching and power measurement, have been minimized [4], the error in this near-field correction is considered to be a significant contributor to the total measured gain uncertainty. This study also considers different operating frequencies, different horn sizes and different horn-to-horn separation distances.

2. HORN GAIN PRODUCT AND ITS NEAR FIELD CORRECTION

In the three-antenna measurement technique employed at CSIRO National Measurement Laboratory, the gain of a horn antenna is determined by measuring the transmission loss versus separation between two horns. And, in a typical laboratory environment with a finite measurement separation distance, a near-field correction is generally required to obtain the far-field true gain.

The gain product of the transmitting and receiving antennas can be expressed by the ratio of transmitting and receiving powers as

$$G_T G_R = (\frac{4\pi R}{\lambda})^2 \bullet \frac{P_R}{P_T} \qquad R \to \infty \tag{4}$$

By the Lorentz reciprocity theorem which allows the calculation of an antenna radiation field in either transmitting or receiving case, the power ratio is [3]

$$\frac{P_R}{P_T} = A\left|\int_{S_1}\int_{S_2} E_1(x,y) \bullet E_2(\xi,\eta)\frac{\exp(-jkr)}{r}ds_1 ds_2\right|^2 \tag{5}$$

where A is an R-independent factor, and S_1 and S_2 are the apertures of the transmitting and receiving horn apertures, respectively. $E_2(\xi,\eta)$ is the receiving horn aperture field distribution which can be expressed similarly to eq.(1) as,

$$E_2(\xi,\eta) = E_2 \cos(\frac{\pi\xi}{a})\exp\{-jk[\delta_\xi(\xi)+\delta_\eta(\eta)]\} \tag{6}$$

The distance r between arbitrary points (x, y) and (ξ, η) in both horn apertures can be approximated for separations much greater than the aperture dimensions, by

$$\begin{aligned} r &= [R^2 + (x-\xi)^2 + (y-\eta)^2]^{1/2} \\ &\approx R + \frac{(x-\xi)^2 + (y-\eta)^2}{2R} \end{aligned} \tag{7}$$

For ease of discussion, we consider both horns to be of identical geometry. The phase term in eq. (5) can be expressed as

$$\begin{aligned} \Phi &= -jk[r(x,y,\xi,\eta) + \delta_x(x) + \delta_y(y) + \delta_\xi(\xi) + \delta_\eta(\eta)] \\ &\approx -jk[R + \frac{(x-\xi)^2 + (y-\eta)^2}{2R} + \frac{x^2+\xi^2}{2l_h} + \frac{y^2+\eta^2}{2l_e}] \end{aligned} \tag{8}$$

where l_e and l_h are radii of phase curvature on the aperture, and each could be assigned any constant value between the axial and slant lengths in E- and H- planes, respectively.

In eq. (8), the space phase term $\frac{(x-\xi)^2 + (y-\eta)^2}{2R}$ becomes insignificant as the separation between the two horns becomes infinite. As we move in from Fraunhofer region to the Fresnel region, this space phase term contributes more to the total phase term, and it makes the wave front appear more curved. So if a measurement is performed in the Fresnel region, the space phase term is not negligible, and needs to be included in the correction to the measured near-field horn gain product $G_T(R)G_R(R)$ in order to obtain the far-field horn gain product G_TG_R, which is distance independent.

Substituting eqs.(1), (6) and (8) in eq. (5) and considering both far and near field conditions discussed above, the near-field correction factor $f(R)$ for horn gain product can be expressed as

$$f(R) = \frac{G_T(R)G_R(R)}{G_TG_R}$$

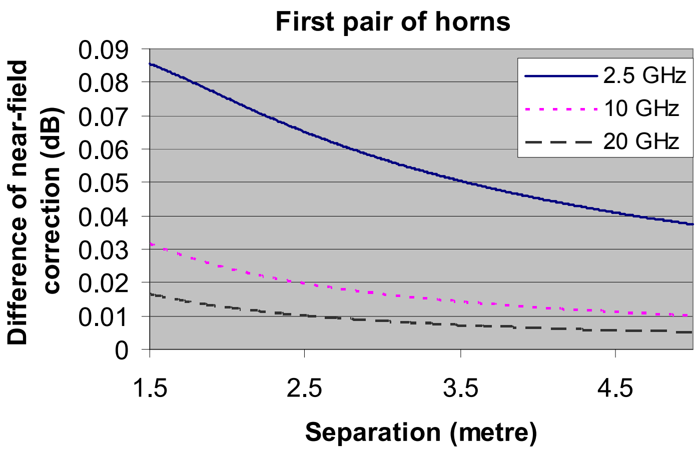
$$= \frac{\left| \int_{S_1}\int_{S_2} \cos(\frac{\pi x}{a})\cos(\frac{\pi \xi}{a})\exp\{-jk[\frac{(x-\xi)^2+(y-\eta)^2}{2R} + \frac{x^2+\xi^2}{2l_h} + \frac{y^2+\eta^2}{2l_e}]\}ds_1 ds_2 \right|^2}{\left| \int_{S_1}\int_{S_2} \cos(\frac{\pi x}{a})\cos(\frac{\pi \xi}{a})\exp\{-jk[\frac{x^2+\xi^2}{2l_h} + \frac{y^2+\eta^2}{2l_e}]\}ds_1 ds_2 \right|^2} \quad (9)$$

which is a function of frequency, horn geometry and separation distance R. Simplified expressions of this correction factor using Fresnel integrals can be found in references [2] and [3].

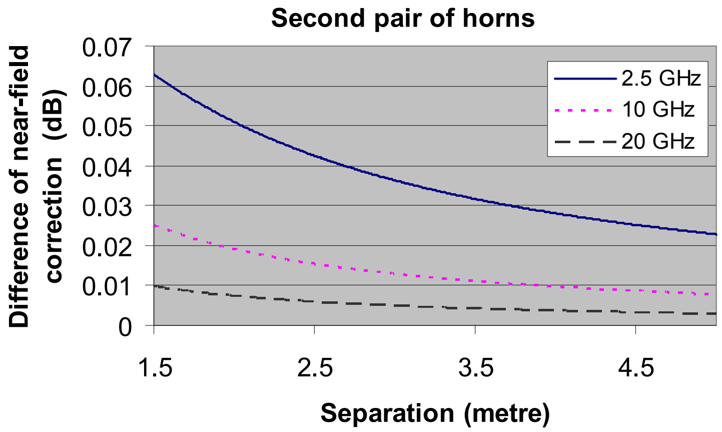
3. RESULTS AND DISCUSSION

Calculations based on eq. (9) were made for two pairs of horns with the same flare angles in both E- and H-plane. The wavelength-normalized dimensions of the first pair of identical horns were: a=3.888, b=2.880, and slant lengths were: l_{hs}=5.352, l_{es}=4.848; and of the second pair were: a=3.24, b=2.40, and l_{hs}=4.46, l_{es}=4.04.

Differences between near-field corrections $f(R)$ obtained using l_s or l_a as the radius of curvature of the phase front in both E- and H-planes versus horn separation for various frequencies are plotted in Fig. 2. By comparing curves for different frequencies in Fig. 2(a) or Fig. 2(b), it is seen that the differences are larger for lower frequencies at the same separation for wavelength-normalized horns. By comparing curves of the same frequency in Figs. 2(a) and Fig. 2(b), it is seen that the differences are larger for the first pair of horns which have bigger apertures. These results indicate that, fundamentally, larger uncertainty associated with near-field correction is likely for lower frequency horn gain measurements at shorter separation distances.



(a)



(b)

Figure 2 The difference between the near-field corrections obtained using l_s or l_a.
Calculated at 2.5 GHz, 10 GHz and 20 GHz.

Calculations were also made for two identical 10 GHz horns having aperture dimensions, a = 19.44 cm, b = 14.41 cm. Such horns are employed as standard gain horn for the calibration of other antennas. Three near-field corrections were obtained when the radii of phase curvature on the aperture used slant lengths, l_{es} = 32.03 cm, l_{hs} = 34.23 cm, axial lengths l_{ea} = 31.21 cm, l_{ha} = 32.82 cm and intermediate lengths, l_e = 31.51 cm, l_h = 33.33 cm, respectively. Fig. 3 shows measured gain products after applying these numerically calculated near-field corrections. The maximum difference between gain products was calculated to be 0.05 dB at 1.5 metre separation and 0.02 dB at 4.5 metre separation. This difference may be regarded as an estimate of the error in $f(R)$ arising from the above approximations of phase curvature variation across the horn aperture.

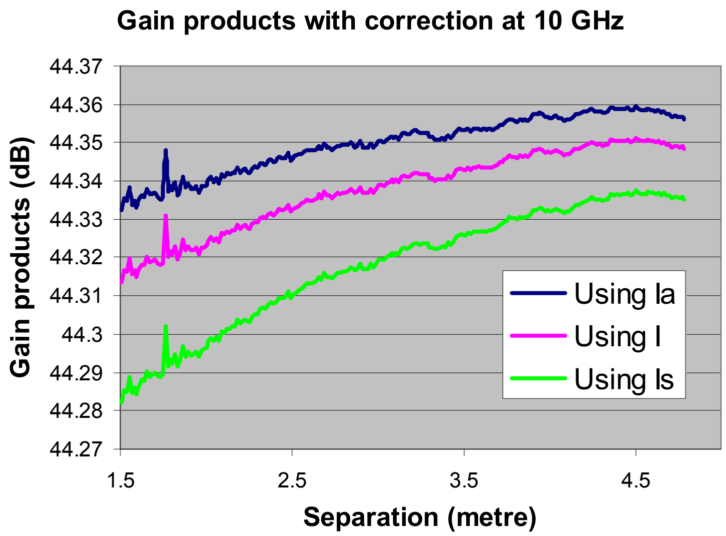


Fig. 3 Measured gain products with near-field correction at 10 GHz.

When such differences in the corrected gain products would be considered negligible in antenna measurements, it is unimportant whether slant or axial length is used to describe the radius of phase curvature in the horn aperture plane. However, for national measurement institutes working to uncertainties of 0.1 dB or less, such differences would be significant [4].

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