PHASE RETRIEVAL OF SCATTERED FIELDS

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1 ABSTRACT

This paper describes an initial investigation of the problem of phase retrieval of fields which have been disturbed by a scatterer. The two techniques of successive projections and conjugate gradients are taken from the research area of near field antenna characterisation and adapted for the problem at hand. Both techniques are discussed and then implemented and compared in a quantitative fashion.

2 INTRODUCTION

As the frequency of electromagnetic radiation increases the measurement of phase information progressively becomes both more difficult and financially more costly. Indeed in the millimetre and submillimetre wave regions a lack of available technology makes the use of amplitude only measurements mandatory for many applications. For this reason phase retrieval from amplitude only measurements of electromagnetic fields is of significant use. Indeed the near field characterisation of antennas from phaseless data has been an area of active and productive research for some years. This paper will discuss the application of iterative phase retrieval techniques to the measurement of scattered fields. To date little to no effort has been applied to the retrieval of phase information from otherwise known fields that have been disturbed by a scatterer of unknown constitution (for example it may not be known if it is dielectric or conductive). Yet this problem has several useful applications including but not limited to the measurement of material dielectric properties, sample thickness estimation, nondestructive testing and inverse scattering. This paper shall present two iterative phase retrieval algorithms that solve this problem, it shall then compare the results of these algorithms and discuss their general usefulness. First however, a brief discussion shall be given on the experimental setup about which these algorithms have been designed.

3 EXPERIMENTAL SETUP

Within this paper phase retrieval is considered for the experiment described by Fig. 1 and implemented through the use of the program CST Microwave Studio. This experiment consists of an incident field which may take any form so long as it is well characterised (both amplitude and phase are known) and it is incident upon a scatterer for which the only assumed knowledge is the object's physical extent (only needs to be known approximately). Though,the scatterer may have any electromagnetic properties, dielectric targets have been used in this paper to facilitate fast and efficient generation of the forward scattering data. The amplitude of the field present on

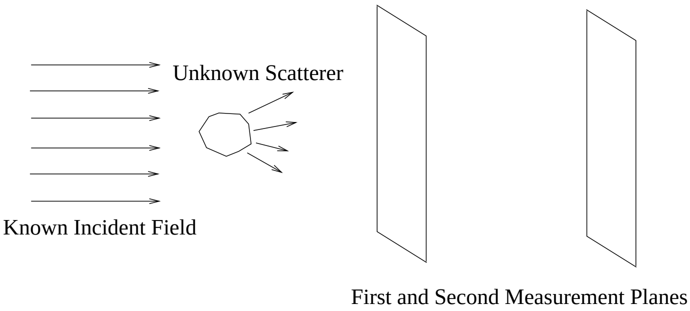


Figure 1: Experimental setup.

the far side of the scatterer is measured at two parallel two dimensional measurement planes separated by a distance of the order of a few wavelengths. With this limited information the algorithms discussed in the coming sections are able to reliably reconstruct the phase at the measurement planes. This problem has similarities with other more developed applications for phase retrieval. The problem has particular similarities to the retrieval of phase information for the nearfield characterisation of antennas. For this reason two algorithms have been taken from this field and adapted to the problem at hand. These algorithms are the well known and commonly used technique of successive projections (similar to the Gerchberg Saxton and error reduction techniques) [1, 2] and the newer conjugate gradients technique [3]. As an additional contribution the authors note that to the best of their knowledge no comparison has yet been made between these two techniques.

4 SUCCESSIVE PROJECTIONS

This is an iterative technique that has been successfully applied to numerous phase retrieval problems. In our particular scenario it functions as follows:

1. Start with the amplitude measured at the first plane and an initial guess for the phase (usually the phase of the undisturbed incident field).
2. Propagate back via Fourier theory to a third plane drawn parallel to the two measurement planes and placed in the middle of the scatterer (the scatterer's plane). Replace the field values outside the scatterer's physical extent with the corresponding values of the known incident field.
3. Propagate via Fourier theory to the second measurement plane where the phase remains unchanged but the amplitude is changed to that measured across the plane.
4. Back propagate once more to the scatterer's plane and change all values outside the scatterer's extent to the corresponding incident field values.
5. Forward propagate back to the first measurement plane and change the amplitude to that measured across the plane.
6. Repeat steps 2-5 until the phase is reconstructed to a suitable accuracy.

This algorithm reliably reconstructs the phase even for surprisingly low signal to noise ratios. However it relies heavily on the assumption that the scatterer is constrained in a spatial extend and that this constraint is known at least loosely. This restraint is important for two reasons, firstly it restrains the possible solutions to those for which the scattered field component originates from the region of the scatterer. Secondly, it means that some of the incident field will pass the scatterer unhindered, thus creating a scenario where the incident field dominates at the extremes of the measurement planes and the scattered field progressively becomes more dominant as one moves towards the centre of the plane. This attribute allows the algorithm to quickly lock onto the correct phase at the extremes of the measurement planes and progressively lock onto the phase as it gets closer to the centre of the plane, thus guiding the solution towards the correct result and providing a useful means of false minima avoidance. For many applications, however, the scatterer's physical extent may either not be known or may be larger than the field probing it (this scenario shall later be referred to as an unconstrained scatterer or a scatterer of infinite extent). For these cases the above algorithm will usually fail as there are no longer strict enough restraints on the solution to guarantee the avoidance of false minima. To overcome this problem the algorithm is adjusted as follows:

1. Start with the amplitude measured at the first plane and an initial guess for the phase (usually the phase of the undisturbed incident field).
2. Filter out all the high spatial frequency data leaving only the extremely low spatial frequencies.
3. Propagate to the second measurement plane and change the amplitude to that measured across the plane.
4. Filter out all the high spatial frequency data leaving only the extremely low spatial frequencies.
5. Back Propagate to the first measurement plane and set the amplitude to that measured across the plane.
6. Repeat steps 2-5 until the rate of change of phase values decreases to a predetermined level.
7. Once the above mentioned rate of phase change is reached, the limit imposed upon the spatial frequencies is increased and steps 2-6 repeated.
8. Steps 2-7 are repeated until the full domain of spatial frequencies are included in the algorithm and the phase is reconstructed to a suitable accuracy.

This adaption allows the algorithm to first latch onto the phase contribution from low spatial frequency data before progressively solving for the contributions from higher spatial frequencies. This significantly improves the algorithm's ability to avoid false minima by first reconstructing the general shape of the phase distribution and then progressively reconstructing finer and finer detail. While this technique is similar to that used for antenna measurement, there are a number of differentiating factors aside form the application. These include:

- In the antenna measurement case the physical extent restraint is applied to the antenna aperture dimensions.
- The antenna measurement scenario is simplified slightly by the absence of an incident field.
- In the scattering case the unhindered passage of the incident field improves false minima avoidance and provides a logical starting estimate for the phase.
- Scatterers of infinite extent must be catered for but infinite antennas need not be.
- The progressive use of higher spatial frequency terms is not utilised by antenna measurement techniques.

5 CONJUGATE GRADIENTS

The conjugate gradients technique also operates in an iterative manner, this time according to the following steps:

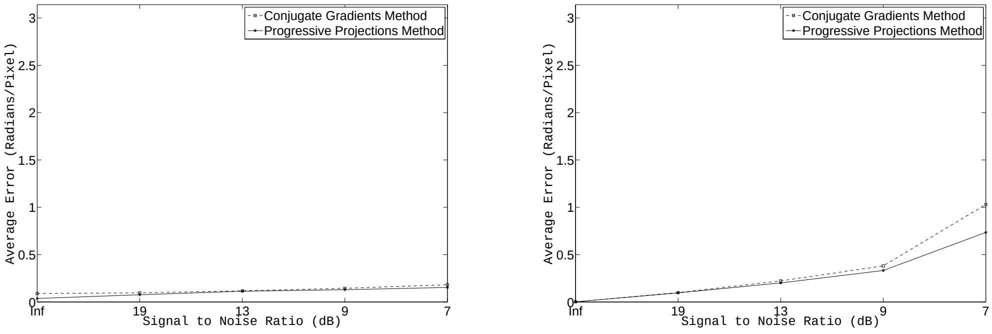
1. Starting at the scatterer's plane (as defined in the previous section) an initial guess is made of the transmitted field by back propagating the measured field at the first plane and the incident field's phase. This field is characterised with as small yet lossless a set as possible so as to maximise the data/unknowns ratio.

2. The estimate is up sampled and then padded with the incident field so that it has the same sampling rate and physical extent as the sampled data.
3. This field is then propagated to the first and second planes where a cost function is evaluated that compares the power of the current estimate of the field at the measurement planes to the measured power.
4. The gradient of this cost function with respect to the field estimate at the scatterer's plane is found using Fourier theory. Using this gradient a new search direction and step size is found via the conjugate gradients method. Due to the simplicity of the cost function the step function is evaluated analytically.
5. The field estimate at the scatterer's plane is then updated using the evaluated search direction and step size.
6. Steps 2-5 are repeated until the phase is reconstructed to a suitable accuracy.

Like the successive projections technique this approach utilises a physical extent restraint. This is done to maximise the data to unknowns ratio as stated in point 2 above. Unfortunately for many applications (with beam widths or measurement planes that are narrower than the scatterer's extent) the scatterer is effectively infinite in extent, for these applications a similar approach is taken to that of the successive projections technique:

1. Starting at the scatterer's plane an initial guess is made of the transmitted field in the same manner as before except no restraint is applied to the physical extent.
2. The estimate is up sampled to the same rate as the measured data (no padding is required as no restraint exists on the scatterer's physical extent).
3. All the high spatial frequency data are filtered out leaving data only for the extremely low spatial frequencies.
4. This field is then propagated to the first and second planes where a cost function is evaluated that compares the power of the current estimate of the field at the measurement planes to the measured power.
5. The gradient of this cost function is found using Fourier theory. Using this gradient a new search direction (limited by the low pass filter of step 3) and step size is found via the conjugate gradients method.
6. The field estimate at the scatterer's plane is then updated using the evaluated search direction and step size.
7. Steps 2-6 are repeated until the rate of change of phase values decreases to a predetermined level.
8. Once the above mentioned rate of phase change is reached, the limit imposed upon the spatial frequencies is increased and steps 2-7 repeated.
9. Steps 2-8 are repeated until the full domain of spatial frequencies is included in the algorithm and the phase is reconstructed to a suitable accuracy.

As with the successive projections technique this adaption allows the conjugate gradients algorithm to avoid false minima by first reconstructing the general shape of the phase distribution through the low spatial frequency terms before progressively including the higher spatial frequency terms and reconstructing the image detail. The distinctions that exist between the successive projections technique defined here and its antenna measurement equivalent also apply to the conjugate gradient technique. In addition to these differences it is also useful to note that in the antenna measurement case [3] the conjugate gradient algorithm assumes that the antenna has a high directivity thus reducing the number of terms required to represent the unknowns. This assumption is not valid for a scatterer but fortunately the presence of an incident field and the resulting false minima avoidance allows this approach to reliably reconstruct the phase without limiting the scatterer's directivity.



(a) For a spatially constrained scatterer.

(b) For a spatially unconstrained scatterer.

Figure 2: Error versus signal to noise ratio for both the conjugate gradients and successive projections techniques.

6 RESULTS

To investigate the performance of these two algorithms the scenario of Fig. 1 was simulated through the CST Microwave Studio program. A plane wave source was used as the incident field into which a scatterer, of dimensions 4*4*2 wavelengths and consisting of a range of varying dielectric and conductive regions, was placed (with the shortest edge parallel to the direction of propagation). Any incident field could have been used, however, a plane wave was chosen to simplify the forward scattering process. The first measurement plane was placed 4 wavelengths from the scatterer while the second plane was placed a further 10 wavelengths from the first. Both planes measured 14*14 wavelengths in size and collected amplitude measurements at the Nyquist rate. These measurements were then subjected to white noise to simulate measurement uncertainty. This experiment was repeated for a range of signal to noise ratios and then for a similar scatterer which completely obscured the measurement planes' view of the incident field. For each reconstruction the average error was measured in terms of the mean error in radians per pixel. Fig. 2(a) and Fig. 2(b) show the quantitative results of these experiments. From these figures it is evident that both techniques perform well when reconstructing the phase from both constrained and unconstrained scatterers. Neither technique has a significant problem reconstructing the phase until small signal to noise ratios are applied to the case of an unconstrained scatterer. In this case both techniques begin to show significant errors with the successive projections technique outperforming the conjugate gradient technique. There are also other important factors concerning these two algorithms which should be considered by the reader prior to choosing one to implement:

- The conjugate gradient method unlike successive projections has a cost function which is guaranteed to decrease monotonically and this is of significant use when determining a stopping criterion.
- The successive projections approach is (for the example above) roughly a factor of two times faster than the conjugate gradients approach. This efficiency difference will vary depending on the size of the problem and the nature of the scatterer. While more investigation is required to determine how the efficiencies change, the successive projections approach will for most (if not all) scenarios be more efficient computationally.
- The conjugate gradients approach has a much stronger theoretical founding then the originally ad hoc design of successive projections. This allows for detailed and simplistic theoretical analysis of local minima avoidance.
- While the successive projections approach is intuitive and ad hoc, considerable theoretical work has been

done to demonstrate its abilities and limitations [4].

- The intuitive nature of the successive projections algorithm allows it to adapt more easily to additional a prior knowledge and to other specialised tailoring then the conjugate gradients approach.
- The successive projections algorithm is simpler to implement and the input parameters simpler in nature and more logical then conjugate gradients'.
- The performance comparison done here is for scattered fields and other phase retrieval applications (e.g. antenna characterisation) may favour the two algorithms differently.

Due to the simulated nature of the experiments performed for this paper, the dimensions of the scatterer and measurement planes are relatively small. In the near future nonsynthetic measurements should be made to investigate behaviour for problems of a larger size.

7 CONCLUSION

This paper has taken two techniques used within the field of nearfield antenna characterisation from amplitude only data and has adjusted them for the problem of phase retrieval for scattering experiments. The adjustments made for this new application were discussed and it was demonstrated that significant changes were required to adjust these algorithms to the new problem. A quantitative comparison was used to demonstrate the performance of the two algorithms across a range of noise values and with both physically constrained and unconstrained scatterers. It was shown that for a constrained scatterer high performance was achieved by both algorithms even at low signal to noise ratios. For the case of an unconstrained scatterer both algorithms performed well for large to moderate signal to noise ratios, with their error rates becoming significant for lower signal to noise ratios. To the authors knowledge this is the first published comparison of the conjugate gradients and successive projections algorithms. This comparison has shown that the two algorithms perform equally well except for the case of an unconstrained scatterer with a small signal to noise ratio, under which condition the successive projections algorithm performed best. Other non performance based differences between the two approaches were also discussed and it was noted that the performance comparison of this paper is only valid for the case of phase retrieval for scattered fields. Future work in this area shall include a more detailed comparison of the two techniques both against signal to noise ratios and their capability for false minimum avoidance as well as the implementation of the algorithms on nonsynthetic data.

References

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