

# OPTIMISATION OF A LUNEBURG LENS FED BY A CORRUGATED HORN ANTENNA

Nasiha Nikolic<sup>(1)</sup>, John S. Kot<sup>(2)</sup>

<sup>(1)</sup> *CSIRO ICT Centre, Radiophysics Laboratory, PO Box 76, Epping, NSW 1710, AUSTRALIA*

*E-mail: [Nasiha.Nikolic@csiro.au](mailto:Nasiha.Nikolic@csiro.au)*

<sup>(2)</sup> *As above, but E-mail: [John.Kot@csiro.au](mailto:John.Kot@csiro.au)*

## ABSTRACT

Low profile Luneburg lenses (LL) fed by a horn antenna have been recently used for a variety of airborne applications. To achieve the required gain for some specific applications, usually an array of the hemispherical lenses is used. The hemispherical lenses, mounted on a conducting ground plane, are fed by the horn sources which can be pivoted about the lenses. The combination of the rotating ground plane and the pivoting sources provides a substantial 3-D coverage that can be used to track the position of the targeted communications satellite. So far, the standard circular horns were used to feed the lens [1]. In this paper, we describe a theoretical method which may be used for optimisation of the lens-horn system.

## 1 INTRODUCTION

The Luneburg lens is a spherical dielectric lens with the property that energy from a feed source at any point on the spherical surface propagates through the sphere to emerge as a collimated beam on the other side of the sphere. An ideal Luneburg lens antenna consists of a dielectric sphere with varying permittivity, ranging gradually from two at the centre to one at the lens surface. In practice, the Luneburg lens is constructed as a radially uniform multi shell spherical lens that efficiently focuses the feed energy. However, in order to achieve the high antenna characteristics, a large number of shells may be required. Instead of increasing the number of the layers, the high performance of the lens may be achieved by optimizing the lens profile, ie by making a non-uniform lens and optimizing the thickness and relative dielectric permittivity of each layer. In [2], the Genetic Algorithm (GA) was used to optimize a non-uniform and two shell lens antenna to achieve high gain and low-side lobe levels. In the optimization, they assumed that the lens is fed by a standard horn antenna or an open-ended waveguide. The source antenna was modelled by using an end-fire antenna consisting of four infinitesimal dipole. This is not very accurate representation of the source. Also, the GA was used to optimize the lens profile only, while the parameters of the source antenna were not optimized.

In this paper we present a theoretical method which may be used for the optimization of the whole system, ie. both the lens profile and the source horn are optimized. Also, the horn antenna is modelled more accurately by using the equivalent currents at the horn aperture. The theoretical model of the source may be improved further by calculating the field distribution across the horn aperture by using commercially available software packages, such as MWS [3].

In the analysis, we first represent the horn by a linear combination of the spherical harmonics [4]. The source horn is modelled assuming that the lens is not present, ie. the mutual coupling between the lens and the horn is neglected. Once the source is represented by the spherical wave expansions, we can calculate the scattered field due to the lens and the gain of the lens-horn system. Then, the lens parameters (radii and dielectric permittivities of each layer) may be optimized for the highest gain by using the GA. This way, we obtain the highest gain with a chosen horn antenna. To obtain the highest gain in general, we change the size of the horn's aperture, and we re-optimize the lens parameters again. After a few iterations the optimum dimensions for the feed horn and the lens profile are obtained.

## 2 SOURCE REPRESENTATION

### 2.1 The Taylor expansion

In this Section, the electric and magnetic fields are expanded as linear combinations of the spherical harmonics, following the method described in [4]. As described in [4], the electromagnetic field may be represented by two expansions: the Taylor and the Laurent expansions. The Taylor expansion for the electromagnetic fields is valid within

the region limited by the ‘maximum’ sphere. The ‘maximum’ sphere is the largest sphere (with radius  $R_{\max}$  as shown in Fig. 1) concentric with the spherical wave expansion origin that does not intercept any part of the source current distribution. The Laurent expansion is valid outside the minimum sphere,  $R_{\min}$  shown in Fig. 1, and this expansion is described in the next Section.

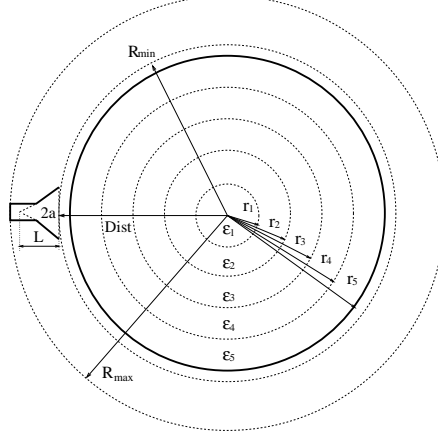


Fig. 1 A Luneburg lens fed by a horn antenna

The electric and magnetic fields inside the ‘maximum’ sphere may be represented by linear combinations of the spherical harmonics as:

$$\begin{aligned}\vec{E} &= \sum_{N=1}^{N_{\max}} \alpha_N \vec{E}_{\pm N} \\ \vec{H} &= \sum_{N=1}^{N_{\max}} \alpha_N \vec{H}_{\pm N}\end{aligned}\quad (1)$$

where the expansion coefficients are calculated from:

$$\alpha_N = K_N \int_{S_i} (\vec{J} \cdot \vec{E}_N - \vec{M} \cdot \vec{H}_N) dS \quad (2)$$

The index  $N$ , the constant  $K_N$  and the electromagnetic fields  $\vec{E}_N$  and  $\vec{H}_N$  are defined in [4, p36]. The integral in (2) is calculated over the horn’s aperture surface.  $\vec{J}$  and  $\vec{M}$  are the equivalent electric and magnetic currents at the horn’s aperture. According to [4], the empirical estimate for the number of modes in (2) is:

$$N_{\max} = 1.5 k R_{\max} \quad (3)$$

where  $k = 2\pi/\lambda$  is the wave phase constant in free-space. To calculate the coefficients  $\alpha_N$  in (2), we assume that the equivalent electric and magnetic current distributions on the horn’s aperture are:

$$\begin{aligned}\vec{J} &= -\hat{x} \frac{1}{\eta} J_0(k_\rho \rho) \\ \vec{M} &= -\hat{y} J_0(k_\rho \rho)\end{aligned}\quad (4)$$

where  $\hat{x}$  and  $\hat{y}$  are standard Cartesian coordinates,  $\eta$  is free space impedance,  $k_\rho$  is the smallest zero of the Bessel function  $J_0(k_\rho \rho) = 0$ , i.e.  $k_\rho a = 2.405$  and  $a$  is the aperture radius [6].

## 2.2 The Laurent expansion

The region for validity of the Laurent expansion is determined by the minimum sphere,  $R_{\min}$ , see Fig. 1. The minimum sphere is the smallest sphere concentric with the spherical wave expansion origin that does not intercept any part of the source current distribution. The electric and magnetic fields outside the minimum sphere are represented as:

$$\begin{aligned}\vec{E} &= \sum_{N=1}^{N_{\max}} \beta_N \vec{E}_N \\ \vec{H} &= \sum_{N=1}^{N_{\max}} \beta_N \vec{H}_N\end{aligned}\quad (5)$$

where the spectrum coefficients  $\beta_N$  are specified by

$$\beta_N = K_N \int_{S_i} (\vec{J} \cdot \vec{E}_{\pm N} - \vec{M} \cdot \vec{H}_{\pm N}) dS \quad (6)$$

The empirical estimate for the largest radial index spectrum coefficients is [4]:

$$N_{\max} = k R_{\min} \quad (7)$$

## 3 SCATTERING BY THE LUNEBERG LENS

Once the electric and magnetic fields radiated by the source are represented in terms of the spherical harmonics we may calculate the far field and the gain of the lens-horn system. The far field and the gain are calculated by using the theoretical method described in [7]. Here we repeat only the main steps of this method.

First, the total electromagnetic field exterior to the lens is represented as a sum of the known source radiation incident on the lens and the unknown scattered radiation. Since we are calculating the incident field on and inside of the lens, we represent the incident field by the Taylor linear expansion (1). Then, we assume similar representations for the scattered electric and magnetic fields outside of the lens and in each layer of the lens. In these expansions we use the spherical harmonics  $\vec{E}_{\pm N}$  as in the Taylor series, and some unknown coefficients. To calculate the unknown coefficients in the expansions we apply:

- 1) the boundary conditions at the lens surface,
- 2) the boundary conditions at the interface between each consecutive dielectric layer of the lens and
- 3) the condition for the finiteness for the electromagnetic fields at the origin.

The application of all boundary conditions yields the system of linear equations which is solved for the unknown coefficients in the expansions for the scattered field.

Once the coefficients in the expansions for the scattered fields are calculated, the total radiated field and the gain are calculated as the sum of the scattered field and the source field. The contribution of the source field to the total far field is included by representing the source field in terms of the coefficients  $\beta_N$ , calculated from the Laurent expansion (5).

## 4 RESULTS

The system of a Luneburg lens fed by a corrugated horn antenna was optimised by using the Genetic Algorithm (GA). We assumed that the lens radius is  $R = 5.75\lambda$  and that the phase centre of the corrugated horn is at the distance  $Dist = 1.1R$  measured from the lens centre. The Luneburg lens, shown in Fig. 1, was approximated with 5 layers. First, the coefficients  $\alpha_N$  and  $\beta_N$  given in (2) and (6) are calculated. In this calculation it is assumed that the horn aperture radius is  $0.6\lambda$ .

The coefficients  $\alpha_N$  are calculated numerically and they are presented in Table 1. As shown in Table 1, these coefficients are increasing rapidly since they depend on the functions  $h_n^{(2)}(x)$ . However, the observation point is inside the maximum sphere, ie  $x' < x$ , and the  $j_n(x')$  functions of the terms  $\bar{E}_{\pm N}$  force rapid overall convergence of the series (1). To check the convergence, the electric field  $\bar{E}$  in (1) is calculated using different number of the spherical harmonics  $N_{\max}$  in (1), and the results are also shown in Table 1.

The numerical results for the coefficients of the Laurent expansion,  $\beta_N$  and the electric field  $\bar{E}$  in (5) are shown in Table 2. The coefficients  $\beta_N$  are decreasing since they depend on the functions  $j_n(x)$  and  $j_n(x) \rightarrow 0$ , for  $n > x$ . However, the functions  $h_n(x')$  of the terms  $\bar{E}_N$  build up rapidly in magnitude for  $n > x'$ . Since, the observation point must be external to minimum sphere ( $x < x'$ ), the spherical wave expansion weights decay more rapidly and overall convergence is ensured in this case as well.

We used the same number of modes for the Taylor and Laurent expansions, although the empirically estimated required number of modes according to (3) and (7) is lower for the Laurent expansions than for the Taylor expansion.

Table 1. Coefficients for Taylor series, calculated using equation (2).

$N_{\max}$	$\text{Re}(\alpha_N)$	$\text{Im}(\alpha_N)$	$\text{Re}\left(\sum_{N=1}^{N_{\max}} \alpha_N \bar{E}_{\pm N}\right)$	$\text{Im}\left(\sum_{N=1}^{N_{\max}} \alpha_N \bar{E}_{\pm N}\right)$
1	0.0862792	0.0689664	0.0000176	-0.000022
10	-0.006857	0.0095521	-0.0038809	-0.007037
20	0.0022333	0.0044904	-0.0120231	-0.002834
30	-0.002338	-0.000453	-0.0084501	-0.007905
40	0.0004299	0.0014995	-0.0084506	-0.005407
50	0.0038947	0.0976657	-0.0087160	-0.005346
60	-186.5645	269.14957	-0.0087386	-0.005345
70	-1.174473e7	5.539524e6	-0.0087398	-0.005346
80	-2.19651e12	2.617631e10	-0.0087395	-0.005346

Once the source is represented with spherical wave expansions, we can calculate the gain of a uniform lens fed by the equivalent source. For the uniform Luneburg Lens, we assume that all shells are of equal thickness ( $r_i = i \cdot R/5$  for  $i = 1 \dots 5$ ). The dielectric permittivities of each shell are calculated from the condition that the focus of the lens is at the point  $1.1R$ , measured from the lens centre. The next step is to optimize the dielectric permittivities  $\varepsilon_i$  ( $i = 1 \dots 5$ ) and radii  $r_i$  ( $i = 1 \dots 4$ ) to achieve the highest gain. In the optimization, the radius of the 5<sup>th</sup> layer was fixed to  $r_5 = R = 5.75\lambda$ . The lens profile is optimized by using the GA [5]. The following parameters were used in the GA: maximum number of generations (the number of iterations) = 40, mutation = 0.1, gene index = 16 and the size of the population = 100. The number of iterations was varied from (20 to 100) but the gain of the horn lens system did not change almost at all after 30-40 iterations.

The results for the optimized and the uniform lens are shown in Table 3. The highest gain was obtained for a horn with the aperture radius of  $a = 0.6\lambda$ . The gain of the optimized lens increased by about 1dBi when compared to the gain of a uniform lens fed by the same horn. That corresponds to increase in efficiency of about 20%.

Table 2. Coefficients for Laurent series, calculated using equation (6).

$N_{\max}$	$\text{Re}(\beta_N)$	$\text{Im}(\beta_N)$	$\text{Re}\left(\sum_{N=1}^n \beta_N \bar{E}_{\pm N}\right)$	$\text{Im}\left(\sum_{N=1}^n \beta_N \bar{E}_{\pm N}\right)$
1	0.0431447	0.03446582	-0.0000192	0.000014739
10	-0.0035520	0.00472752	-0.0029016	0.003423514
20	0.0011136	0.00202266	0.0027628	0.004809876
30	-0.0014280	-0.00009314	-0.0010115	0.007287582
40	0.0002782	0.00014688	-0.0004005	0.007559234
50	6.6080749e-8	3.5709653e-7	0.0011045	0.006910001
60	-2.410489e-11	1.6896451e-11	0.0010970	0.006903104
70	-3.617623e-16	-3.8598453e-17	0.0011061	0.006899058
80	-4.307644e-22	-3.6517322e-22	0.0011039	0.006898820

Table 3. The gain and efficiency of a LL fed by a corrugated horn antenna.

$a$	Gain(uniform lens) dBi Efficiency	Gain(optimized lens) dBi Efficiency
0.3 $\lambda$	26.983 38.24%	28.390 52.88%
0.5 $\lambda$	29.575 69.46%	30.781 91.71%
0.6 $\lambda$	<b>29.972</b> <b>76.13%</b>	<b>30.988</b> <b>96.20%</b>
0.7 $\lambda$	30.028 77.10%	30.766 91.39%

## 5 CONCLUSIONS

A method for the optimization of a Luneburg lens fed by a corrugated horn antenna is described. In the method, we used the GA to improve the efficiency of the lens-horn system. The efficiency of the optimized LL fed by a corrugated horn is improved by about 20% compared to the efficiency of a uniform LL fed by the same horn. The same method may be used for the optimization of the LL fed by any other source, provided that the electric and magnetic fields at the aperture of the source are known.

## 6 ACKNOWLEDGEMENT

Nasiha Nikolic is currently enrolled as a part-time PhD student at Macquarie University, <http://www.mq.edu.au>

## REFERENCES

- [1] M.E. Rayner, "A low-profile Luneburg lens airborne GBS antenna", Datron/Transco Inc., <http://www.ecs.umass.edu/ece/allerton/papers1998/GBS/>.
- [2] H. Mosallaei, Y. Rahmat-Samii, "Nonuniform Luneburg and two-shell lens antennas: Radiation characteristics and design optimization", IEEE Transactions on Antennas & Propagation, Vol. 49, No. 1, May 2001.
- [3] CST Microwave Studio, <http://www.cst.com>.
- [4] P.J. Wood, "Reflector antenna analysis and design", John Wiley & Sons, Inc., 1989.
- [5] Y. Rahmat-Samii, E. Michielssen, "Electromagnetic optimization by Genetic Algorithms", John Wiley & Sons, Inc., 1999.
- [6] T.S. Bird, "An introduction to aperture antennas", Notes prepared for a lecture course, October 1986.
- [7] J. Sanford, "Scattering by Spherically Stratified Microwave Lens Antennas", IEEE Transactions on Antennas & Propagation, Vol. 42, No. 5, May 1994.