THREE DIMENSIONAL MODEL FOR PROPAGATION IN THE TROPOSPHERE AND INVERSE DIFFRACTION

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ABSTRACT

The parabolic wave equation model (PEM) can be applied to model the propagation of electromagnetic signals in the troposphere and electromagnetic scattering. In this paper the concept of inverse diffraction is investigated for the case of the three dimensional (3D) PEM. Inverse diffraction is an algorithm that produces field convergence by inverting the PEM diffraction term and can be applied to find the transmitter location. The inverse diffraction radiolocation algorithm can be related to existing, standard radiolocation methods and is an efficient method for locating a transmitter using intercepted emissions. The 3D PEM presented in this paper gives correct spatial phase. Correct spatial phase for the PEM is essential for the inverse diffraction algorithm to process observed data correctly. Examples are discussed for propagation with atmospheric refraction and over irregular terrain.

INTRODUCTION

The PEM has been applied in underwater acoustics [1], geophysical applications [2] and radio wave propagation [3]. Recent progress with the PEM allows the efficient modelling of radio frequency propagation in irregular terrain, vegetation, and urban areas and under all weather conditions in either two or three dimensions. Using back propagation methods the potential of the PEM is being recognised for treating electromagnetic scattering problems also.

A new application for the PEM, radiolocation using inverse diffraction, has been investigated in [4], [5] and [6]. Also in [6] and [7] a new propagation model related to the PEM, the Huygens' Principle Model (HPM) has been introduced that gives enhanced performance including a wide-angle propagation capability. The PEM and HPM is shown in [6] and [7] to give consistent diffraction field amplitude performance, but some versions of the PEM that do not use the sine transform and instead use the FFT alone such as in [8] can give incorrect spatial phase. The incorrect phase is in the form of cusp in the unwrapped spatial phase profile at specified ranges from the transmitter, instead of the correct quadratic form near the antenna bore-sight. Incorrect spatial phase will in general be undesirable for a propagation model, and will give incorrect radiolocation for inverse diffraction algorithms that are used to process intercepted emissions.

In the development of the 3D PEM, the authors of this paper have used the robust and efficient split-step method [9], [10] for both vertical and horizontal processing. The results presented for inverse diffraction in [4], [5] and [6] are based on the 2D PEM. An extension to the 3D PEM is provided in this paper. Other authors have discussed the 3D PEM see [11].

Outlining the paper, we first formulate the 3D PEM and discuss its properties. Then inverse diffraction algorithms are defined and related to traditional radiolocation methods. A description of results obtained with the 3D PEM is given next. In particular, inverse diffraction for atmospheric refraction and irregular terrain are considered. Finally conclusions about the 3D PEM and inverse diffraction are provided.

PROPAGATION MODEL

Derivations of the PEM beginning with Maxwell's equations subject to boundary conditions have been published previously and will not be repeated here, see [3]. The 3D PEM provides a solution, $u(x, y, z)$, that can be related to the electromagnetic field (transverse tangential magnetic for vertical electric polarisation and transverse tangential electric for horizontal polarisation) where x, y and z are the horizontal range, horizontal cross-range and the vertical altitude coordinates, respectively.

In the split-step algorithm the solution is initialised at zero range using the relation between the normalised far-field pattern of an antenna and its aperture distribution. It is then stepped forward in range as required. Given the field solution (i.e., $u(x, y, z)$ at range x), this paper will provide 3D PEM equations that specify the propagated field solution $u(x+\Delta x, y, z)$ at the extended range of $x+\Delta x$. Diffraction and the environmental factors of refraction, surface reflection and irregular terrain are included in the model. Notation used in this paper is specified below.

- θ, ϕ are elevation, azimuth propagation angles measured from the horizontal, vertical plane, respectively.
- $k = 2\pi / \lambda$ is the free-space wave number, where λ is the wavelength.
- $p_z = k\sin\theta$, $p_y = k\sin\phi$ is the vertical, horizontal spatial frequency spectrum for $u(x, y, z)$.
- $R(x, y, z) = \exp(ik\Delta x(m(x, y, z)-1)/2)$ is the refraction function, $m(x, y, z) = n(x, y, z) + z/a$ is the modified refractive index that takes into account the curvature of the earth, $n(x, y, z)$ is the refractive index of the atmosphere and a is the radius of the Earth. The cross-range distances are assumed small enough to allow the curvature of the Earth to be ignored in cross–range. Note, $i = \sqrt{-1}$.
- $M(x, y, z)$ is the irregular terrain mask function that is either 0 or 1 for below or above the terrain surface, respectively.
- $D(p) = \exp(i\Delta x(\sqrt{k^2 - p^2} - k))$ is the wide-angle PEM diffraction function, see [12]. Note, $p = p_z$ or p_y.

For the 3D PEM the algorithm is decoupled into the vertical and horizontal planes at each range step as

$$u(x+\Delta x, y, z) = R(x, y, z)M(x, y, z)T_V^{-1}[D(p_z)T_V[u(x, y, z)]] \text{ for vertical plane } y = y_j \quad (1)$$

$$u(x+\Delta x, y, z) = M(x, y, z)T_H^{-1}[D(p_y)T_H[u(x, y, z)]] \text{ for horizontal plane } z = z_j \quad (2)$$

It can be seen that this 3D propagation model can represent horizontal cross-range and range-dependent variations in the environment. For vertical plane processing (1) the upper boundary condition is handled by means of the usual window function that is commonly employed with the PEM, such as the Hanning window. For horizontal plane processing (2) the outer boundary conditions are handled also via a suitable window function.

For vertical plane processing the transform T_V is the mixed Fourier transform from the z domain to the p_z domain and T_V^{-1} is its inverse, see [12]. The mixed Fourier transform consists of a linear transform together with the sine transform and ensures that the surface boundary conditions for a given transmitter polarisation are satisfied. For a perfectly conducting surface T_V can be replaced by simply the sine transform. For horizontal plane processing the obvious choice for transform T_H would appear to be the FFT. However, this unfortunately gives incorrect spatial phase as the authors have discovered via tests with MATLAB implementations of the 3D PEM. It turns out that the best choice for T_H is the sine transform. Using the sine transform for T_H, undesirable reflections might be expected from one outer boundary. Fortunately, such reflections are suppressed by the already mentioned window function for the horizontal plane outer boundary conditions.

The simple mask function $M(x, y, z)$ is included in (1) and (2) to model irregular terrain and this does a surprisingly good job. If desired, improved surface reflection modelling can be applied for both vertical and horizontal plane processing with more sophisticated algorithms. Development of these improved methods is currently in progress.

The 3D PEM has a heavy computational requirement and to implement large examples on a PC requires use of the disk during processing. The decoupled algorithm of (1) and (2) helps facilitate this. Given in the form of (1) and (2), 2D PEM versions can reduce the computational requirement significantly.

DSTO Edinburgh has tested the propagation loss predictions and spatial phase behaviour of the PEM against trial data obtained for a wide range of environmental conditions, and the observed data give excellent agreement with the model predictions.

INVERSE DIFFRACTION

It is possible to propose radiolocation algorithms via inverse diffraction that use the PEM, see [4], [5] and [6]. Given all the field information for a vertical or horizontal line (2D PEM) or a vertical plane (3D PEM) at a given range from the transmitter antenna, the forward propagation transformation can be inverted to provide the transmitter location – inverse diffraction. The theoretical concept of inverse electromagnetic problems has been considered by other authors see [13], and inverse methods have been used successfully in many other areas such as geological exploration, image processing, inverse filtering and deconvolution see the website [14].

Perfect inversion of the propagation model (1) and (2) is not possible because information is lost as the PEM solution is stepped forward. However, inversion that is sufficient to locate the transmitter accurately and recover other propagation information is feasible. This can be achieved simply for (1) and (2) by using (1) and (2) again, stepping backwards in range with a sign change of the argument of the exponential term in both $D(p)$ and $R(x, y, z)$. Since perfect inverse propagation is not possible the algorithm is called inverse diffraction – a reference to the fact that the diffraction term has been inverted for both vertical and horizontal processing in (1) and (2).

Inverse diffraction can be related to traditional radiolocation methods such as direction of arrival (DOA), time difference of arrival (TDOA) or differential Doppler (DD). These traditional methods are described in [15]. For the inverse diffraction algorithm to be most effective and given that the PEM models paraxial propagation in the troposphere, intercepted data for radiolocation are needed along a linear path, typically more than 1 km in length and approximately perpendicular to the DOA. For the case of a single transmitter, the key field information required is the quadratic spatial phase profile for the observed field data to be inverted. This spatial phase profile is related to the distance between the transmitter and the receiver. The required spatial phase data for the inverse diffraction algorithm can be obtained in a number of ways including the traditional radiolocation methods mentioned above.

Inverse diffraction algorithms can handle the case of single or multiple transmitters if required. In the general case of multiple transmitters with irregular terrain and an atmosphere, received signal power and spatial phase are both needed for inverse diffraction. The spatial phase data can again be obtained in several ways including the traditional methods of radiolocation. Inverse diffraction is an algorithm that could provide additional capability for these existing radiolocation methods. The additional capability that could be provided is the ability to treat the case of multiple transmitters and environmental conditions.

In many cases of radiolocation using inverse diffraction, knowledge of the irregular terrain via the term, $M(x, y, z)$, can be omitted from the inversion without causing significant error in the estimate of transmitter location. Achieving radiolocation of an acceptable accuracy without requiring knowledge of the irregular terrain may often be advantageous.

The authors are currently investigating the implementation of inverse diffraction for use in radiolocation. One possibility could be using integrated information from DOA, TDOA and DD methods. The concept of inverse diffraction offers the capability of estimating the location (i.e., both DOA and range) of a transmitter via intercept operations conducted within a localised area. The receiver configuration of intercepted data being obtained along a linear track approximately perpendicular to the DOA (as in inverse diffraction) appears to be advantageous in simplifying radiolocation, compared with a more generic operational scenario. One example of how this method may have operational advantages is that the highly complex TDOA algorithm could then be simplified. Another example is when measuring the receiver location, etc is difficult due to the Global Positioning System (GPS) not being available, alternative navigation and timing should be easier to achieve within a localised area of operation. Locating interfering transmitters in the GPS band is being considered as an application for new radiolocation methods such as inverse diffraction, given the sensitivity to interference and widespread application of GPS [16].

In the next section the 3D PEM is used to provide simulation test data for 2D PEM inverse diffraction algorithms. The inverse diffraction algorithm is implemented in 2D PEM mode either in a horizontal or vertical plane to allow faster data processing compared with a full 3D PEM inversion. In a horizontal plane inverse diffraction can provide cross-range and range information, while in a vertical plane transmitter height and range information is obtained. Results are presented with the 3D PEM used for forward propagation and the 2D PEM used for the inversion algorithm. This is an illustration of how the 3D PEM can be used to provide a test capability that would be difficult to achieve in other ways.

RESULTS

Refraction

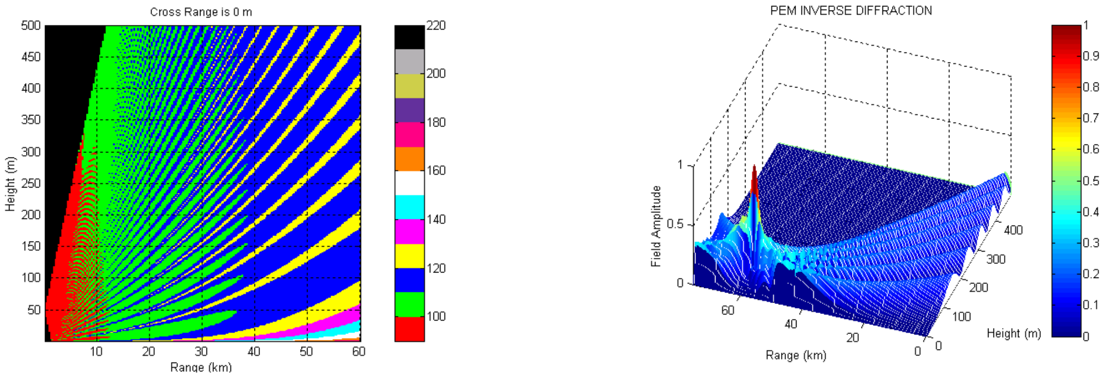
Anomalous propagation can affect the performance of military equipment operating in coastal regions of Australia with both enhanced propagation and radio holes. Refraction and ducts are described in [17], and [18] discusses anomalous propagation in the Great Australian Bight. Recent DSTO Edinburgh monitoring of an L-band transmitter at Price, South Australia, has shown enhancements for significant time periods of around 50 dB over a 65 km sea, surface propagation path. The simulation results discussed in this section show that inverse diffraction is feasible in the presence of refractive effects in the troposphere.

In Figure 1(a) forward propagation for the 3D PEM is shown with a vertical propagation loss coverage diagram on the transmitter antenna boresight at 0 m cross-range. In Fig 1(a) the refractive conditions are a standard atmosphere, the frequency is 3 GHz, the transmitter antenna height is 50 m, the transmitter antenna is omni-directional, the polarisation is vertical and the surface is sea. The maximum range is 60 km, the maximum cross-range is 1 km either side of the transmitter antenna and the maximum vertical height is 500 m. Inverse diffraction applied to receiver data obtained along a vertical linear path between heights of 0 and 500 m at a range 60 km is shown in Fig. 1 (b). In Fig. 1 the bend in the pattern of surface reflection fringes is due to the combined effect of atmospheric refraction and the curvature of the Earth's surface. The estimate of transmitter range obtained from inverse diffraction is 58.2 km, and the estimate for transmitter height is 46.9 m. If inverse diffraction is applied in a horizontal rather than a vertical plane, with receiver data obtained for a 2 km cross-range linear path at 50 m in height, then the range estimate is 60 km with a cross-range estimate of 0 m. Tests have also been run for strong evaporation and surface-based ducts where the duct properties vary with cross-range. In the presence of these ducts inverse diffraction in the vertical plane is sometimes successful but generally is more difficult than for a standard atmosphere. However, good radiolocation in the presence of cross-range variable ducts can be obtained using inverse diffraction in the horizontal plane. For the 60 km range example, radiolocation error is less than 300 m in range and 10 m in cross-range. For the 2 km cross-range the variation in duct top height used was 20 to 40 m for the evaporation duct and 150 to 250 m for the surface-based duct. Inverse diffraction in the horizontal plane has the advantage that the refractive index profile of the atmosphere is not required to obtain good radiolocation.

Irregular Terrain

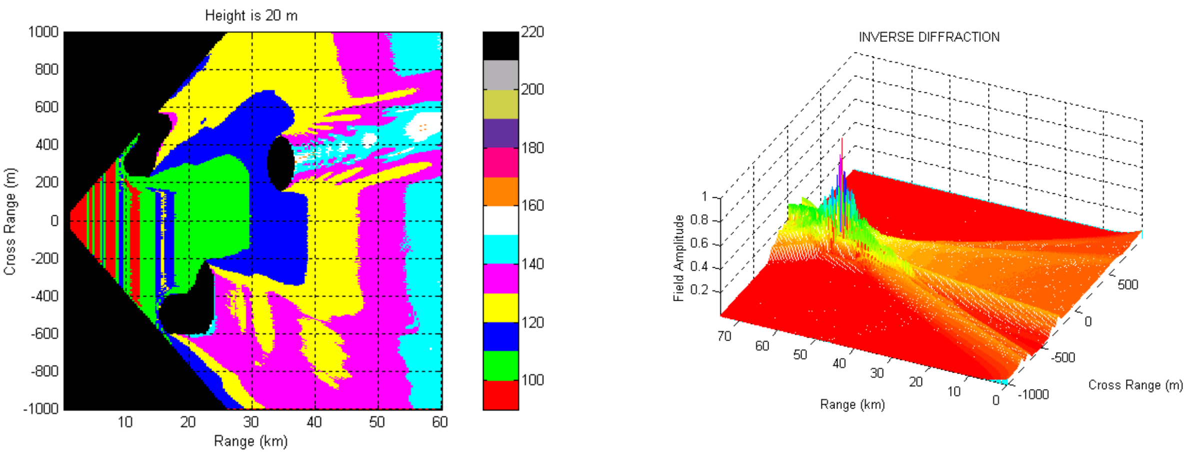
Using the 3D PEM with a standard atmosphere, Fig. 2 (a) shows forward propagation at 3 GHz in a horizontal plane for irregular terrain in the form of 3 "hills" with a maximum height of 80 m. The transmitter is at 0 m cross-range and 50 m height, polarisation is vertical, and the transmitter antenna is omni-directional. The receiver data for inverse diffraction are obtained at 60 km range for a horizontal linear path of length 2000 m at a height of 50 m. For the inversion shown in Fig. 2 (b) knowledge of the irregular terrain or the 3 "hills" is not provided, i.e., inversion is performed with a free-space 2D PEM algorithm. The resulting inversion estimates transmitter range as 60.6 km, and cross-range as 0 m. When a sufficient part of the intercept receiver data are obtained with a good line-of-sight to the transmitter then knowledge of the irregular terrain is not needed.

Simulations have also been done to investigate if inverse diffraction can locate transmitters when the intercepted data are obtained only within the diffraction region. Tests have been done over a 60 km range with knife-edges placed at 30 km. Inverse diffraction in the vertical plane alone has difficulty locating the transmitter. However, operation in a horizontal plane gives good results without knowledge of the irregular terrain. For intercept data obtained in the diffraction region, the use of a full 3D PEM inversion with knowledge of the irregular terrain may be needed in some cases to obtain the required radiolocation accuracy. In such cases it is hoped that fast, simplified algorithms can be devised that will improve radiolocation performance if required. This is a possible area for future research.



(a) (b)

Fig. 1. Forward propagation coverage (propagation loss in dB) in a vertical plane using the 3D PEM is shown in (a) for a standard atmosphere. Frequency is 3 GHz and transmitter antenna height is 50 m. PEM inverse diffraction in the vertical plane is shown in (b) and a peak is obtained that indicates the transmitter height and range.



(a) (b)

Fig. 2. Forward propagation coverage (propagation loss in dB) in a horizontal plane using the 3D PEM is shown in (a) with irregular terrain. Frequency is 3 GHz and the transmitter antenna cross-range is 0 m and height 50 m. PEM inverse diffraction in a horizontal plane is shown in (b) and a peak is obtained that indicates the transmitter cross-range and range.

CONCLUSION

A 3D PEM for propagation in the troposphere has been presented. It has been found that correct spatial phase is obtained only if the correct transform is used in the PEM. PEM inverse diffraction can be used for radiolocation and gives new insights for traditional methods of radiolocation. Results have been presented where the 3D PEM is used to test the radiolocation performance of inverse diffraction algorithms implemented in both the horizontal and vertical plane. It is hoped that the PEM methods discussed in this paper can be used to develop improved radiolocation methods as well as give an excellent modelling capability for propagation in the troposphere.

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