Radio Wave Propagation Algorithms Based on the Reciprocity Principle

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ABSTRACT

The reciprocity principle is best known as a result concerning the interchangeability of antennas. In its general form, however, it can be used to derive a large number of integral results that include the better known integral equations of electromagnetism. Furthermore, numerous extensions form the basis of some quite general approaches to problems in radio wave propagation. The current paper describes some recent developments in this area and discusses the extension of these ideas to non isotropic propagation media through the concept of pseudo-reciprocity.

THE BASIC RECIPROCITY RESULT

Consider an electromagnetic field $\left(\underline{E}_A, \underline{H}_A\right)$ that satisfies the time harmonic Maxwell equations

$$\nabla \times \underline{H}_A = j\omega\varepsilon_A \underline{E}_A + \underline{J}_A \tag{1}$$

and

$$\nabla \times \underline{E}_A = -j\omega\mu_A \underline{H}_A \tag{2}$$

where $\underline{J}_A$ represents the sources of the field, μ_A is the permeability of the propagation medium and ε_A is the permittivity of this medium. Furthermore, consider another unrelated field $\left(\underline{E}_B, \underline{H}_B\right)$ that satisfies the time harmonic Maxwell equations

$$\nabla \times \underline{H}_B = j\omega\varepsilon_B \underline{E}_B + \underline{J}_B \tag{3}$$

and

$$\nabla \times \underline{E}_B = -j\omega\mu_B \underline{H}_B \tag{4}$$

where $\underline{J}_B$ represents the sources of the field, μ_B is the permeability of the propagation medium and ε_B is the permittivity of this medium. From a standard vector field identity

$$\nabla \cdot \left(\underline{E}_A \times \underline{H}_B - \underline{E}_B \times \underline{H}_A\right) = \underline{H}_B \cdot \nabla \times \underline{E}_A - \underline{E}_A \cdot \nabla \times \underline{H}_B - \underline{H}_A \cdot \nabla \times \underline{E}_B + \underline{E}_B \cdot \nabla \times \underline{H}_A \tag{5}$$

Substituting (1) to (4) into (5), and assuming $\mu_A = \mu_B$ and $\varepsilon_A = \varepsilon_B$, we obtain

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$$\nabla \cdot \left(\underline{E}_A \times \underline{H}_B - \underline{E}_B \times \underline{H}_A\right) = \underline{E}_B \cdot \underline{J}_A - \underline{E}_A \cdot \underline{J}_B \tag{6}$$

and , integrating over a volume V that contains all the field sources, the result

$$\int_S \left(\underline{E}_A \times \underline{H}_B - \underline{E}_B \times \underline{H}_A\right) \cdot d\underline{S} = \int_V \left(\underline{E}_B \cdot \underline{J}_A - \underline{E}_A \cdot \underline{J}_B\right) dV \tag{7}$$

where S is the surface of volume. We consider the sources to be bounded and the media tend to free space at infinity. As the surface S to tends to infinity, we will have the electric and magnetic fields satisfying the relations $\underline{H} \approx \hat{\underline{r}} \times \underline{E}/\eta_0$ and $\hat{\underline{r}} \cdot \underline{E} \approx 0$. Consequently, the integrand on the left hand side of (7) will be $O\left(r^{-3}\right)$ and the integral will tend to zero. We therefore obtain the relation

$$\int_V \underline{E}_B \cdot \underline{J}_A dV = \int_V \underline{E}_A \cdot \underline{J}_B dV \tag{8}$$

which is the standard form of the Lorentz Reciprocity result [1]. If fields A and B are generated by antennas A and B respectively, and both driven by current I, then (8) implies that voltage V_B induced in antenna A by current I in antenna B equals voltage V_A induced in antenna B by current I in antenna A (i.e. $V_A = V_B$). This form of reciprocity implies that the antennas at each end of a communications link may be interchanged without effect, even if those antennas are different.

PROPAGATION TECHNIQUES BASED ON RECIPROCITY

Now consider the derivation of the reciprocity result with one field $(\underline{E}, \underline{H})$ caused by sources outside the closed surface S and the other $(\underline{E}_0, \underline{H}_0)$ caused by an electric dipole element with current $\underline{J}_0$ at a position $\underline{r}_0$ within surface S . Result (7) now becomes

$$\underline{E}(\underline{r}_0).\underline{J}_0 = \int_S (\underline{E} \times \underline{H}_0 - \underline{E}_0 \times \underline{H}).d\underline{S} \tag{9}$$

Furthermore, if $(\underline{E}_0, \underline{H}_0)$ is now a dipole field generated by a magnetic current element $\underline{M}_0$ at position $\underline{r}_0$, then

$$-\underline{H}(\underline{r}_0).\underline{M}_0 = \int_S (\underline{E} \times \underline{H}_0 - \underline{E}_0 \times \underline{H}).d\underline{S} \tag{10}$$

These equations [3] are variants of the normal reciprocity result [1] and constitute one of the many integral formulations of electromagnetism. Although the material properties of the medium do not appear explicitly, the equations will still apply to non homogeneous media. The inhomogeneous effects are incorporated by the fact that the dipole fields satisfy the Maxwell equations for the non homogeneous medium. If the sources of field $(\underline{E}, \underline{H})$ are bounded, the above equations will still apply if S is an open surface that extends to infinity and separates the sources from the point $\underline{r}_0$. If the fields are known on surface S, the above equations can be used to extend the solution out from this surface. As a consequence, if only forward propagation is significant, equations (9) and (10) can be used to advance an electro-magnetic field onwards from a surface S on which it is known. Obviously, to turn the above equations into a propagation algorithm, we need a suitable dipole solutions and exact solutions are available for only a few non homogeneous media. As a consequence, we tend to use approximate solutions, normally one of the geometric optics (GO) variety. These, however, will only have a limited range of validity and so equations (9) and (10) can only be used to develop the solution to a limited range. We can, however, use (9) and (10) to advance the fields to a new surface at a range where the approximate solution is valid and repeat the process until sufficient coverage has been achieved.

For many propagation problems the direction of propagation is approximately horizontal and, in such cases, the above integral equations can be approximated as [4]

$$\underline{E}(\underline{r}_0).\underline{J}_0 = -2\int_S \frac{\underline{E} \cdot \underline{E}_0}{\eta} dS \tag{11}$$

where, when there are no ground reflections, the GO dipole field takes the form

$$\underline{E}_0 \approx \frac{j\omega\mu}{4\pi} \underline{J}_0 \frac{\exp(-j\beta\phi)}{s} \tag{12}$$

with β is the free-space wave-number, ϕ is the phase distance between the source and test point $\underline{r}$ and s is the spreading distance. The above approach has been considered in references [2-4] and has yielded simulations that agree with the more usual parabolic approach to such problems.

Fig. 1 below shows the results of some simulations using the above techniques. These results show the propagation losses in a 30m high atmospheric duct with antenna placed at a height of 15m. The results show a significant amount of energy to be trapped in the duct.

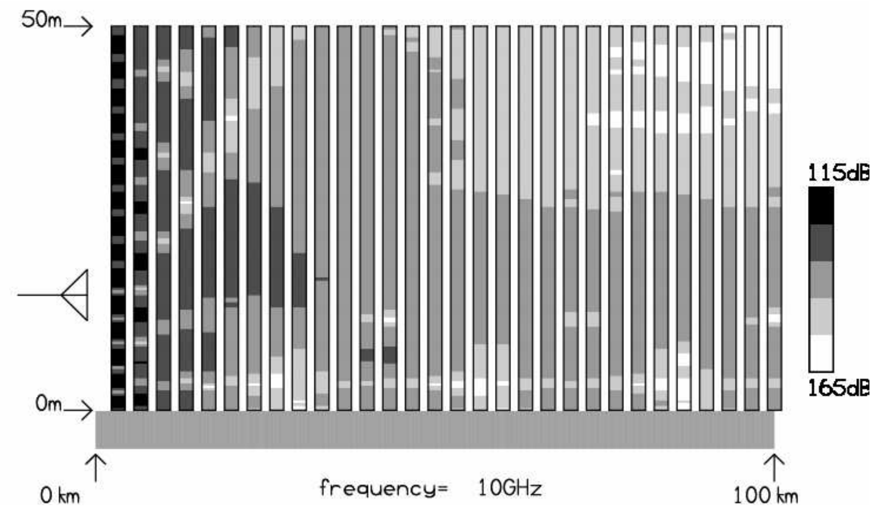

Figure.1 Propagation losses in a 30m high duct with antenna at a height of 15m.

EXTENSIONS TO ANISOTROPIC MEDIA

It is interesting to note that the equations (9) to (11) will still hold for anisotropic media (the permittivity and permeability are now expressed as tensors) providing that $\varepsilon_B=\varepsilon_A^T$ and $\mu_B=\mu_A^T$ (note that the superscript T indicates the transpose of a tensor). As a consequence, the techniques described in the previous section will still be viable providing the dipole GO fields are calculated using the relevant anisotropic permittivity and permeability. The extension is fairly straightforward and so we will not dwell any further on it. Instead, we will look at what happens when the anisotropic medium is perturbed. With reference to the A and B fields of section I, consider propagation media that are related through $\varepsilon_B=\varepsilon_A^T+\delta\varepsilon$ and $\mu_B=\mu_A^T+\delta\mu$ ($\delta\varepsilon$ is a tensor permittivity perturbation and $\delta\mu$ is a tensor permeability perturbation). On substituting (1) through (4) into (5),

$$\nabla\cdot\left(\underline{E}_A\times\underline{H}_B-\underline{E}_B\times\underline{H}_A\right)=\underline{E}_B\cdot\underline{J}_A-\underline{E}_A\cdot\underline{J}_B+j\omega\left(\underline{H}_A\delta\mu\underline{H}_B-\underline{E}_A\delta\varepsilon\underline{E}_B\right) \qquad (13)$$

Integrating this relation over a volume V that contains all the field sources, we obtain

$$\int_S\left(\underline{E}_A\times\underline{H}_B-\underline{E}_B\times\underline{H}_A\right)\cdot d\underline{S}=\int_V\left(\underline{E}_B\cdot\underline{J}_A-\underline{E}_A\cdot\underline{J}_B\right)dV+j\omega\int_V\left(\underline{H}_A\delta\mu\underline{H}_B-\underline{E}_A\delta\varepsilon\underline{E}_B\right)dV \qquad (14)$$

where S is the surface of volume V. As the surface S tends to infinity, the integrand on the left hand side tends to zero (assuming the medium becomes isotropic at infinity) and we obtain the relation

$$\int_V\underline{E}_B\cdot\underline{J}_A dV=\int_V\underline{E}_A\cdot\underline{J}_B dV-j\omega\int_V\left(\underline{H}_A\delta\mu\underline{H}_B-\underline{E}_A\delta\varepsilon\underline{E}_B\right)dV \qquad (15)$$

If $\delta\mu=\delta\varepsilon=0$ we have the normal reciprocity relation $\int_V\underline{E}_B\cdot\underline{J}_A dV=\int_V\underline{E}_A\cdot\underline{J}_B dV$ where the material properties of media A and B can be anisotropic, so long as they are related according to $\varepsilon_B=\varepsilon_A^T$ and $\mu_B=\mu_A^T$. We term this result ‘pseudo reciprocity’. Budden [6] has considered such extensions to reciprocity for magneto-ionic media where $\varepsilon_B=\varepsilon_A^T$ is achieved by reversing the background magnetic field. Let $\delta\underline{E}_B$ be the perturbation in electric field B caused by the perturbations in material properties, then

$$\int_V \delta \underline{E}_B \cdot \underline{J}_A dV = -j\omega \int_V \left(\underline{H}_A \delta\mu \, \underline{H}_B - \underline{E}_A \delta\varepsilon \, \underline{E}_B \right) dV \tag{16}$$

For $\underline{J}_A$ an ideal dipole, and fixed permeability, this reduces to

$$\delta \underline{E}_B \cdot \underline{J}_A = j\omega \int_V \underline{E}_A \delta\varepsilon \, \underline{E}_B dV \tag{17}$$

If $\delta\varepsilon$ is proportional to a small parameter, the electric field can be expanded in terms of this paramenter (i.e. $\underline{E}_B = \underline{E}_B^0 + \underline{E}_B^1 + \underline{E}_B^2 + \ldots$) with the terms in the perturbation series satisfying

$$\underline{E}_B^{i+1} \cdot \underline{J}_A = j\omega \int_V \underline{E}_A \delta\varepsilon \; \underline{E}_B^i dV \tag{18}$$

The first perturbation is effectively the Born approximation.

Now consider fields that can be described by GO in their unperturbed state. The major contribution to the integral in the above expressions will arise from the vicinity of the ray path that joins dipole A to the sources of field B. The GO approximation to an unperturbed field is $\underline{E} = A\underline{P} \exp(-j\beta\phi)$ where ϕ can be expanded in terms of a ray orthogonal coordinate system (X_1, X_2) as $\phi = \phi_0 + \phi^{11} X_1 X_1 + 2\phi^{12} X_1 X_2 + \phi^{22} X_2 X_2 + \ldots$ On using such expansions, integral equation (19) will reduce to

$$\delta \underline{P}^B \cdot \underline{\hat{J}}_A = \frac{\omega^2 \mu}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{s_{AB}} \underline{P}^A \delta\varepsilon \left(\underline{P}^B + \delta \underline{P}^B \right) \sqrt{\left| \Phi^+ \right|} \exp\left(-j\beta \left(\phi_A^{IJ} + \phi_B^{IJ} \right) X_I X_J \right) \frac{ds}{N} dX_1 dX_2 \tag{19}$$

where $\underline{\hat{J}}_A$ denotes the normalized current and matrix Φ^+ has elements $\phi_A^{ij} + \phi_B^{ij}$. In the case of small perturbations, and an isotropic medium, (21) is effectively the expression developed by Gherm et al. [7]. Expression (21), however, provides a general integral equation for developing the general perturbed field out from the sources of field B and small perturbations are only one of many possible solution strategies. When the length scale of the perturbations is large,

$$\delta \underline{P}^B \cdot \underline{\hat{J}}_A = \frac{-j\omega^2 \mu}{2\beta} \int_0^{s_{AB}} \underline{P}^A \delta\varepsilon \left(\underline{P}^B + \delta \underline{P}^B \right) \frac{ds}{N} \tag{20}$$

A simple application of (22) is the derivation of an expression for the rotation of the polarization vector in the ionosphere caused by the magnetic field of Earth. In this case $\varepsilon_{ij} = \varepsilon_0 \left(N^2 \delta_{ij} + \frac{j}{\beta} \tau_{ij} \right)$ where $\tau_{ij} = -\tau_{ij}$, $N^2 = 1 - X$, $\tau_{12} = -\beta Y_3 X$, $\tau_{13} = \beta Y_2 X$, and $\tau_{23} = -\beta Y_1 X$ where $X = \omega_c^2 / \omega^2$, $\underline{Y} = -(\beta_H / \beta)(\underline{B}_0 / B_0)$, ω_c is the plasma frequency, ω_H is the gyro frequency, ω is the wave frequency, $\beta_H = \omega_H / c$ and $\underline{B}_0$ is the background magnetic field of the Earth. Consider a background ionosphere without the magnetic field of Earth and for which the each ray is planar. Choose $\underline{\hat{J}}_A$ to be perpendicular to this plane then $\underline{P}^A = \underline{\hat{J}}_A$ and

$$\delta \underline{P}^B \cdot \underline{\hat{J}}_A = \frac{1}{2} \int_0^{s_{AB}} \underline{\hat{J}}_A \mathrm{T} \left(\underline{P}^B + \delta \underline{P}^B \right) \frac{ds}{N} \tag{21}$$

where matrix T has elements τ_{ij}. This provides a correction to the background electric field to account for the effect of Earth's magnetic field and is exactly the expression that would arise from GO theory.

The extended version of reciprocity also allows us to generalize equation (11) to the situation of a medium with non isotropic perturbations. Equation (11) will now takes the form

$$\underline{E}(\underline{r}_0).\underline{J}_0 = -2\int_S \frac{\underline{E}\cdot\underline{E}_0}{\eta}\,dS + j\omega\int_V \underline{E}\,\delta\varepsilon\,\underline{E}_0\,dV \qquad (22)$$

This equation can be used as a basis for the study of propagation through a disturbed non isotropic medium (the ionosphere for example) by generalization of the ideas in section 2.

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