

SPATIAL AND TEMPORAL IONOSPHERIC MAPPING WITH OUTLIER AND MISSING SAMPLES

Mike D.E. Turley and Robert S. Gardiner-Garden

Defence Science and Technology Organisation, Edinburgh, SA, 5111, Australia.

Email: mike.turley@dsto.defence.gov.au

Abstract

Accurate ionospheric mapping has application in real time coordinate transformation of HF skywave radar targets to ground coordinates. Typically Kriging methods are used to spatially interpolate derived parameters from a sparse network of ionospheric sounder observations. Here we modify the model by making use of the current sounder observations to form a sample spatial covariance matrix. A statistical non-homogeneity detection test is introduced for selecting robust stationary and homogenous observations. In this way spatial and temporal outliers are rejected. The formulation also provides a means to handle missing observations.

I. INTRODUCTION

Smoothly mapping ionospheric parameters from a non-uniformly spaced set of measurement sites to a regular grid is required in many data modelling problems. Here we are concerned with the case of providing a real-time ionospheric model (RTIM) [1] for applications such as coordinate registration in the Jindalee Operational Radar Network (JORN) HF skywave radar. In this situation coordinate registration is the act of transforming the position of a target detected in radar coordinates to ground position. To do this an accurate description of the HF skywave propagation path and the state of the ionospheric electron density is required in real time. A means to do this is to form a parametric model of the ionosphere based on real time data from the non-uniformly spaced vertical incidence sounder network positioned sparsely across the Australian continent.

The parameters typically include estimates for each ionospheric layer of height, thickness and critical frequency. Propagation paths are then calculated using electron density height profiles derived from these parameters.

Ionospheric mapping requires interpolation of the parameters between the sounder sites. A means to do this is by forming a spatial (site to site) correlation model for each parameter and performing the interpolation (a methodology referred to as Kriging in the geo-physical literature). This result is only optimal in a LMS sense if the covariance matrix is true. However, the estimate may be improved with use of the sample covariance matrix provided the samples form an homogenous set. A non-homogeneity test is provided to select samples. This same test may be used to address the problem of poor derivation of parameters from sounder measurements (the outlier problem). We also derive a robust solution to the catastrophic failure of sounders (the missing sample problem).

II. IONOSPHERIC MAPPING WITH ALL SITES AVAILABLE

Here we wish to solve the problem of estimating parameter \hat{P}_k (e.g. ionospheric layer height) at location k given measurement of the parameter from a network of M sounder sites

$$\bar{P} = [P_1 \dots P_j \dots P_M]^T$$

where $(\cdot)^T$ denotes the transpose operation. A model describing the parameters is:

$$\bar{P} = \bar{P}_b + \bar{P}_\sigma + \bar{P}_\epsilon \quad (1)$$

where subscripts b , σ and ϵ denote background, geographic variance and measurement noise components respectively. Without loss of generality we remove the background (trend) component from all calculations and recover it as a final step in the estimation process.

The error in the parameter estimate may be calculated as:

$$e_k = P_k - \hat{P}_k$$

where the true parameter value P_k is for now assumed known. Let us estimate the value as a linear combination of the known site parameter values:

$$\hat{P}_k = \bar{\alpha}_k^H \bar{P}$$

where the M dimensional vector $\bar{\alpha}_k$ are the site weights and $(\cdot)^H$ denotes the Hermitian transpose operation. We wish to optimise the weight selections based on the ensemble average $E\{\cdot\}$ of past parameter measurements, so let us introduce the following error cost function:

$$J = E\{|e_k|^2\} = E\{(P_k - \bar{\alpha}_k^H \bar{P})^H (P_k - \bar{\alpha}_k^H \bar{P})\}$$

the optimal minimum variance weights may be calculated by:

$$\arg \min_{\bar{\alpha}_k} J \Rightarrow \frac{\partial J}{\partial \bar{\alpha}_k^*} = \bar{0} = R\bar{\alpha}_k - \bar{q}_k$$

where $(\cdot)^*$ denotes conjugation,

$$R = E\{\bar{P}\bar{P}^H\}$$

and

$$\bar{q}_k = E\{\bar{P}P_k^*\}$$

Hence the optimal site parameter weights are:

$$\bar{\alpha}_k = R^{-1}\bar{q}_k$$

and the parameter estimate at the target location is:

$$\hat{P}_k = \bar{\alpha}_k^H \bar{P} \quad (2)$$

The covariance matrix R may be estimated as a sample correlation matrix \hat{R} from past sounder site measurements provided the sample set is stationary. The independent sample set size should be greater than $2 \times M$ for estimation of the covariance matrix [2] to achieve SNR losses less than 3 dB of optimal performance. However estimation for the site to target spatial correlations \bar{q}_k are unavailable since it depends on the unknown true parameter P_k at the target site.

From (1) with the background trend removed the components of the site parameters contribute to the spatial covariance matrix as:

$$\begin{aligned} R &= E\{(\bar{P}_\sigma + \bar{P}_\epsilon)(\bar{P}_\sigma + \bar{P}_\epsilon)^H\} \\ &= R_\sigma + E\{\bar{P}_\sigma \bar{P}_\epsilon^H\} + E\{\bar{P}_\epsilon \bar{P}_\sigma^H\} + R_\epsilon \\ &= R_\sigma + R_\epsilon \end{aligned} \quad (3)$$

assuming the σ and ϵ components are uncorrelated, which is likely to be true for large geographic scale ionospheric phenomena but false otherwise (when the ϵ is poor the σ component will compensate for the error).

One solution to this problem is to use the Kriging methodology where the spatial covariances R_σ and \bar{q}_k are substituted with an empirical model. For our purposes the site to site covariance relationship could be modelled as a function of the distance between the sites, such as the Gaussian function:

$$R_\sigma = \sigma^2 \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & e^{-\left(\frac{d_{ij}}{d_c}\right)^2} & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \quad (4)$$

or the exponential function $\exp(-(d_{ij}/d_c))$, where i, j denote row and column indices respectively, d_{ij} represents the site separations, d_c is the correlation distance, and σ^2 is the component variance. If desired the model correlations may be specified as separate longitudinal and latitudinal components. The measurement error component of the site parameter values is modelled as additive white Gaussian noise:

$$R_\epsilon = \epsilon^2 I \quad (5)$$

with variance ϵ^2 and I represents the identity matrix.

A. Incorporation of Sample Matrix

Clearly if the modelled spatial correlations R and \bar{q}_k do not represent the true correlations then the Kriging methodology will not be optimal. Alternatively if a sample spatial covariance matrix \hat{R} is available then this should be used providing suitable sample support exists. The sample matrix may be estimated from the set of T recent past sounder measurements (snapshots):

$$\hat{R}(t) = \frac{1}{T} \sum_{i=0}^{T-1} \bar{P}(t-i) \bar{P}^H(t-i)$$

where t denotes the current snapshot index and the background trend is again removed. To avoid rank-deficiency $T \geq M$ and as discussed previously near optimal performance is achieved for $T \geq 2M$. To ensure the samples are representative of the current data a non-homogeneity detection (NHD) test [3] based on the generalised inner product is applied. Define the NHD test statistic for the past T snapshots as:

$$\hat{\mu}(t-i) = \bar{P}(t-i)^H \hat{R}^{-1}(t) \bar{P}(t-i) \quad i = 0, \dots, T-1$$

where the initial estimate for the sample covariance matrix is formed using all L snapshots. The calculated test statistics are sorted and thresholded to keep T_h homogeneous snapshots, then the sample covariance matrix is reformed using the subset of homogeneous snapshots.

In the case of insufficient snapshots ($T_h < 2M$) then a mixture of empirical and sample matrix estimates may be used, such as:

$$R = \left(1 - \frac{T_h}{2M}\right)R_\sigma + \frac{T_h}{2M}\hat{R}$$

Snapshots of site to target correlations \bar{q}_k are unavailable, so instead a correlation model is formed as a constrained polynomial model from R .

III. SOLUTIONS FOR MISSING SITES

Missing sounder measurements occur for a variety of reasons and the situation must be handled to produce a RTIM. It is convenient to provide two formulations to this problem for later reference.

A. Solution 1

The obvious approach when the parameters from one or more sites are unavailable is to exclude the site information from the calculations. Let us formulate this approach by using a site selection matrix S , so that the subset of $L < M$ good sites becomes:

$$\bar{P}_S = S\bar{P}$$

where $L = M - N$ with N equal to the number of missing sites. The selection matrix is composed by way of example ($L = 4$, $M = 5$, with the 3rd site excluded) as:

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In a similar fashion:

$$\bar{q}_S = S\bar{q}_k \quad \& \quad R_S = SR S^H$$

hence the parameter estimate with missing sites becomes:

$$\begin{aligned} \hat{P}_S &= \bar{q}_S^H R_S^{-1} \bar{P}_S \\ &= \bar{q}_k^H S^H (SR S^H)^{-1} S\bar{P} \\ &= \bar{\alpha}_S^H \bar{P} \end{aligned} \tag{6}$$

and the coinciding weight solution is:

$$\bar{\alpha}_S = S^H (SR S^H)^{-1} S\bar{q}_k \tag{7}$$

B. Solution 2

An alternative but equivalent approach is to solve for the weights using cost function J subject to zero gain constraints on the missing site weights:

$$C^H \bar{\alpha}_k = \bar{g} = \bar{0}$$

where the elements of the $M \times N$ dimensional constraint matrix C are:

$$c_{ij} = \begin{cases} 1, & \text{for } i = M_j, \quad j = 1, \dots, N \\ 0, & \text{otherwise} \end{cases}$$

The constraints may be included in the cost function via use of Lagrange multipliers $\bar{\lambda}$:

$$\begin{aligned} J &= E\{(P_k - \bar{\alpha}_k^H \bar{P})^H (P_k - \bar{\alpha}_k^H \bar{P})\} \\ &+ \bar{\lambda} (C^H \bar{\alpha}_k - \bar{g}) + \bar{\lambda}^* (C^T \bar{\alpha}_k^* - \bar{g}^*) \end{aligned}$$

and the optimal weight solution becomes:

$$\bar{\alpha}_k = R^{-1} \left(\bar{q}_k - C (C^H R^{-1} C)^{-1} (C^H R^{-1} \bar{q}_k - \bar{g}) \right) \tag{8}$$

IV. TIME VARYING SITE AVAILABILITY

The availability of individual sounder sites varies independently. As site availability changes the estimation of the RTIM is influenced. Abrupt fluctuations in the RTIM can affect radar operations and in particular coordinate registration of radar tracks. Two situations arise as the sites appear and disappear. As a site disappears less information becomes available resulting in poorer estimation. Preferably the temporal transition should be smooth. The second situation occurs when a new site becomes available. In this situation more information is available potentially leading to estimation of a better RTIM.

A. Constraint Transition

The solution to the missing sounder problem is given in (8) for $\bar{g} = \bar{0}$. This equation is however a general solution providing the non-missing result as well. Let the constraint gains be:

$$\bar{g} = (1 - \gamma)C^H R^{-1}\bar{q}_k$$

then (8) may be written as:

$$\bar{\alpha}_k = R^{-1} \left(\bar{q}_k - \gamma C (C^H R^{-1} C)^{-1} C^H R^{-1} \bar{q}_k \right) \quad (9)$$

so $\gamma = 1$ provides the fully constrained solution and $\gamma = 0$ provides the unconstrained result. The control parameter γ therefore provides a means to transition from the two states, provided an estimate of the missing parameter values are used for all but when $\gamma = 1$. The parameter estimate at current time 't' then becomes:

$$\hat{P}_k(t) = \bar{\alpha}_k^H(t, \gamma(\Delta t))\bar{P}(t)$$

where Δt is the time lapsed since the last measurement, and the measurement vector is given by:

$$\bar{P}(t) = \begin{bmatrix} P_1(t) \\ \vdots \\ P_j(t) \\ \vdots \\ \hat{P}_{m1}(t - \Delta t) \\ \vdots \\ P_M(t) \end{bmatrix} \quad (10)$$

with missing site values replaced by the previous estimate from that site. The control parameter γ would suitably be a transition function such as a sigmoid function with state transit of about 60 minutes on site loss and shorter for site advent.

B. Parameter Estimate Transition

Perhaps a simpler solution is to calculate the parameter estimate at each measurement update according to the number of available sites, for example equations (7, 8), then apply a forgetting factor $\beta(\Delta t)$:

$$\hat{P}_k(t) = \beta(\Delta t)\bar{\alpha}_T(t)^H\bar{P}(t) + (1 - \beta(\Delta t))\hat{P}_k(t - \Delta t)$$

where β transits from $0 \rightarrow 1$ on each change of sounder state.

V. OUTLIERS IN SOUNDER MEASUREMENTS

Ionospheric parameters are derived from the sounder measurements. For example height, thickness and critical frequency from a Vertical Incidence Sounder ionogram. This can introduce outliers through measurement error or the parameter derivation process. The outliers will in turn affect the interpolation calculation (2) through all terms. If an outlier is found then that sounder may be excluded or given reduced weighting as in Sec. IV, with γ in (9). To detect outliers the NHD may again be used. Here the outlier detection statistic is defined as:

$$\hat{\mu}_m = \bar{P}_m^H R_m^{-1} \bar{P}_m \quad m = 1, \dots, M$$

where \bar{P}_m represents the parameter vector for sounder sites excluding site m (see (6)), and similarly R_m is the site to site spatial covariance matrix excluding site m . A sorted plot of $\hat{\mu}_m, m = 1, \dots, M$ will establish outlier sites.

VI. CONCLUSIONS

Accurate ionospheric mapping has application in real time coordinate transformation of HF skywave radar targets to ground coordinates. Typically Kriging methods are used to spatially interpolate derived parameters from a sparse network of ionospheric sounder observations. Here we modified the model by making use of the current sounder observations to form a sample spatial covariance matrix. A statistical non-homogeneity detection test is introduced for selecting robust stationary and homogenous observations. In this way spatial and temporal outliers are rejected. The formulation also provides a means to handle missing observations.

REFERENCES

- [1] R.I. Barnes, R.S. Gardiner-Garden and T.J. Harris, "Real time ionospheric models for the Australian Defence Force," *Workshop on Radio Science Proceedings, WARS 2000*, 2000.
- [2] I.S. Reed, J.D. Mallett and L.E. Brennan, "Rapid convergence rate in adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 10, no. 6, pp. 853–863, 1974.
- [3] M.C. Wicks, W.L. Melvin and P.Chen, "An efficient architecture for nonhomogeneity detection in space-time adaptive processing airborne early warning radar," *IEE RADAR 97*, Oct. 1997.