THE UNIVERSITY OF ADELAIDE

Department of Electrical and Electronic Engineering

RADIO WAVE PROPAGATION ALGORITHMS BASED ON THE RECIPROCITY PRICIPLE

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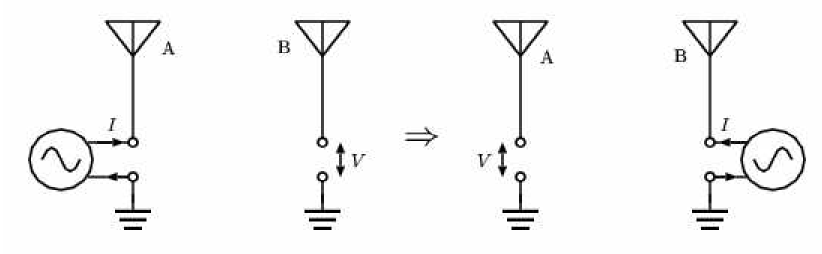
Reciprocity between Antennas

For antennas A and B, interchanging transmitter and receiver does not change received voltage.



General Reciprocity

$$\int_S \left(\underline{E}_A \times \underline{H}_B - \underline{E}_B \times \underline{H}_A\right) \cdot d\underline{S}$$

$$= \int_V \left(\underline{E}_B \cdot \underline{J}_A - \underline{E}_A \cdot \underline{J}_B - \underline{H}_B \cdot \underline{M}_A + \underline{H}_A \cdot \underline{M}\right) dV$$

where volume V has surface S. Field $\left(\underline{E}_A, \underline{H}_A\right)$ has sources $\left(\underline{J}_A, \underline{M}_A\right)$ and $\left(\underline{E}_B, \underline{H}_B\right)$, a quite unrelated field, has sources $\left(\underline{J}_B, \underline{M}_B\right)$. If the sources are the antennas A and B, we obtain the reciprocity result for antennas.

An Integral Equation

The general result yields integral relation

$$\underline{E}\left(\underline{r}_0\right) \cdot \underline{J}_0 = \int_S \left(\underline{E} \times \underline{H}_0 - \underline{E}_0 \times \underline{H}\right) \cdot d\underline{S}$$

when $\left(\underline{E}_A, \underline{H}_A\right)$ results from electric dipole $\underline{J}_o$

$$\underline{H}\left(\underline{r}_0\right) \cdot \underline{M}_0 = -\int_S \left(\underline{E} \times \underline{H}_0 - \underline{E}_0 \times \underline{H}\right) \cdot d\underline{S}$$

when it results from magnetic dipole $\underline{M}_o$. These results also hold for a non homogenous region.

In the paraxial limit, above equations reduce to

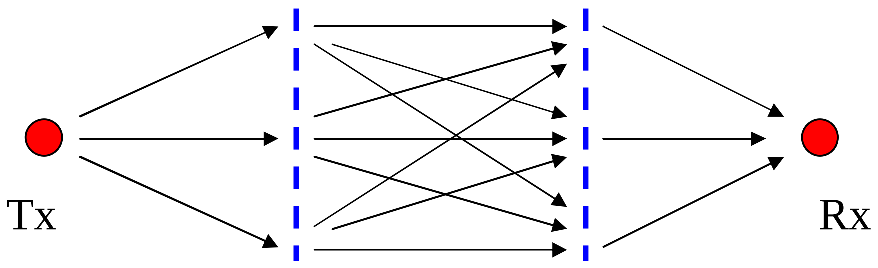
$$\underline{E}(\underline{r}_0)\cdot \underline{J}_0 = -\int_S \frac{2N}{\eta_0}\underline{E}\cdot\underline{E}_0\, dS \quad (1)$$

where S is a surface that is perpendicular to the propagation direction [Coleman, IEEE A&P Trans, 2005]. This can be used to develop field $\underline{E}$ out from surface S. Providing we do not move too far away from S, dipole field $\underline{E}_0$ can be represented by the GO approximation

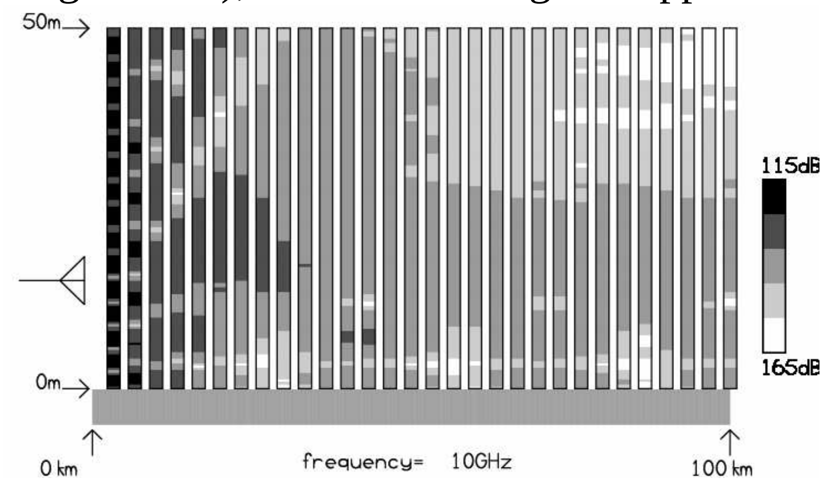
$$\underline{E}_0 = \frac{j\omega\mu}{4\pi}\underline{P}_0 \frac{\exp(-j\beta\phi)}{d} \quad (2)$$

where d and ϕ are the spreading and phase distances from the source. $\underline{P}_0$ is the polarisation vector for the dipole source.

To find the field between Tx and Rx, the region between them is divided by a sequence of surfaces, close enough for (2) to be valid. The field is then developed from surface to surface.



The figure below shows propagation loss in a sea surface duct (duct height=30m and antenna height=15m), calculated using this approach.



The results are identical to those calculated using parabolic equation (PE) approach [Levy, 2004]. The current method, however, does not require a rectangular discretisation mesh and is hence more flexible. Further, it is easily blended with other methods.

Anisotropic Media

Integral equation (1), and its more exact forms, still holds for anisotropic materials provided that the 'reciprocal' fields have different media and the tensors describing their permittivity and permeability satisfy $\varepsilon_0 = \varepsilon^T$ and $\mu_0 = \mu^T$.

If the medium of one of the pseudo reciprocal fields is perturbed, we obtain the general result

$$\int_S \left(\underline{E}_A \times \underline{H}_B - \underline{E}_B \times \underline{H}_A\right) \cdot d\underline{S}$$

$$= \int_V \left(\underline{E}_B \cdot \underline{J}_A - \underline{E}_A \cdot \underline{J}_B - \underline{H}_B \cdot \underline{M}_A + \underline{H}_A \cdot \underline{M}_B\right) dV$$

$$+ j\omega \int_V \left(\underline{H}_A \cdot \delta\mu \cdot \underline{H}_B - \underline{E}_A \cdot \delta\varepsilon \cdot \underline{E}_B\right) dV$$

For $\delta\varepsilon = \delta\mu = 0$, we term this result ***pseudo reciprocity***. For $\underline{J}_A$ an ideal dipole, and $\delta\mu = 0$,

$$\delta \underline{E}_B \cdot \underline{J}_A = j\omega \int_V \underline{E}_A \cdot \delta\varepsilon \cdot \underline{E}_B dV \qquad (3)$$

where $\delta\underline{E}_B$ is the perturbation to the B field caused by a perturbation $\delta\varepsilon$ to its permittivity. If $\delta\varepsilon$ is proportional to a small parameter, $\underline{E}_B = \underline{E}_B^0 + \underline{E}_B^1 + \underline{E}_B^2 + \ldots$ with

$$\underline{E}_B^{i+1} \cdot \underline{J}_A = j\omega \int_V \underline{E}_A \cdot \delta\varepsilon \cdot \underline{E}_B^i dV$$

The first two terms of the expansion constitute the Born approximation in the isotropic case.

An alternative procedure can be used when a GO approximation is acceptable in the unperturbed case. The GO approximation to the unperturbed fields is given by $\underline{E} = A\underline{P}\exp(-j\beta\phi)$. Expanding ϕ in ray orthogonal coordinate system (X_1, X_2)

$$\phi = \phi_0 + \phi^{11} X_1 X_1 + 2\phi^{12} X_1 X_2 + \phi^{22} X_2 X_2 + \ldots$$

For field B, we consider the perturbation to cause a perturbation $\delta\underline{P}_B$ in its polarisation vector.

From (3), we obtain integral equation

$$\delta \underline{P}^B \cdot \underline{\hat{J}}_A = \frac{-\omega^2 \mu}{2\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{s_{AB}} \underline{P}^A \cdot \delta\varepsilon \cdot \left(\underline{P}^B + \delta \underline{P}^B\right)$$

$$\times \sqrt{\left|\Phi^+\right|} \exp\left(-j\beta\left(\phi_A^{IJ} + \phi_B^{IJ}\right) X_I X_J\right) ds\, dX_1 dX_2$$

where $\underline{\hat{J}}_A$ denotes the normalized current, matrix Φ^+ has elements $\phi_A^{ij} + \phi_B^{ij}$ and s_{AB} is the distance between sources of fields A and B. For small perturbations, and an isotropic medium, this is effectively the expression developed by Gherm et al. [Radio Science 2005].

The above result provides an integral equation for developing the general perturbed field out from the sources of field B. Small perturbations are one of many solution methods for this equation.

When the length scale of the perturbations is large, we have the simple 1D integral equation

$$\delta \underline{P}^B \cdot \underline{\hat{J}}_A = \frac{j\omega^2 \mu}{2\beta} \int_{0}^{s_{AB}} \underline{P}^A \cdot \delta\varepsilon \cdot \left(\underline{P}^B + \delta \underline{P}^B\right) ds \qquad (4)$$

Ionospheric Medium

For an ionosphere with magnetic field $\underline{B}_0$

$$\varepsilon_{ij} = \varepsilon_0 \left(N^2 \delta_{ij} + \frac{j}{\beta} \tau_{ij} \right) \quad \text{and} \quad \tau_{ij} = -\tau_{ij}$$

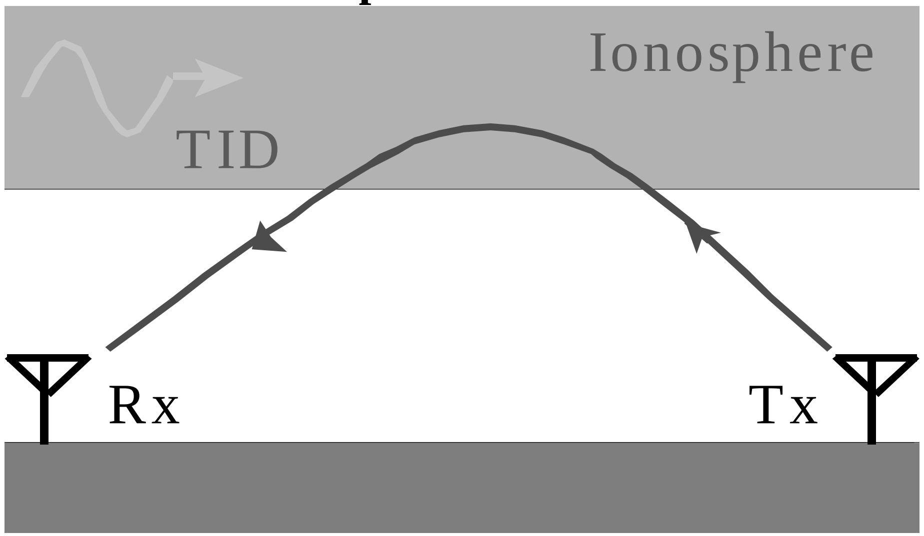
with $N^2 = 1 - X$, $\tau_{12} = -Y_3 X$, $\tau_{13} = Y_2 X$, $\tau_{23} = -Y_1 X$, $X = \omega_c^2 / \omega^2$, $\underline{Y} = -\beta_H \left(\underline{B}_0 / B_0\right)$, ω_c = plasma frequency, ω_H = gyro frequency and ω = wave frequency. NB $\mu_{0ij} = \mu_{ji} = \mu_0 \delta_{ij}$

In calculating the GO solution above, S and ϕ can be obtained by means of standard ray tracing and the rotation angle γ of the polarization vector, by integrating

$$\frac{d\gamma}{ds} = \frac{\beta_H X}{2N} \frac{\underline{B}_0}{B_0} \cdot \frac{d\underline{r}}{ds}$$

Note that, for pseudo reciprocal fields, the rotation will be in opposite directions.

A Disturbed Ionosphere



For a disturbed ionosphere

$$\delta\varepsilon = \varepsilon_0 \delta X\left(-I + j\hat{\mathrm{T}}/\beta\right)$$

where $\hat{T}$ has elements $X^{-1}\tau_{ij}$ and I is the unit tensor. $\underline{P}_A$ and $\underline{P}_B$ are polarisation vectors for the pseudo reciprocal background fields and we obtain convenient equations on expanding in terms of such vectors (i.e. $\delta\underline{P}^B \approx \rho_1\underline{P}^B + \rho_2\underline{P}_\perp^B$ where $\underline{P}_\perp^B$ is orthogonal to the ray and $\underline{P}^B$).

The ρ_I components satisfy

$$\rho_I(s_{AB}) = \frac{\omega^2\mu}{2\pi}\int_0^{s_{AB}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(I_J + \rho_J(s)\right)\delta\varepsilon_{IJ} \times\exp\left(-j\beta\phi^{IJ}X_I X_J\right)\sqrt{|\Phi|}\left(1-\frac{X_K}{N_0}\frac{\partial N_0}{\partial X_K}\right)dX_1 dX_2\frac{ds}{N_0}$$

where $\delta\varepsilon_{IJ}$ are the components of $\delta\varepsilon_{ij}$ in the orthogonal directions that define quantities ρ_I (I_J is 1 when $J=1$ and 0 when $J=2$).

When the length scale of the perturbations is large, this reduces to

$$\rho_I(s_{AB}) = -\frac{j\beta}{2}\int_0^{s_{AB}}\left(I_J + \rho_J(s)\right)\hat{\varepsilon}_{IJ}\delta X_0\frac{ds}{N_0} \qquad (5)$$

Once the background ray tracing has been performed, and the behaviour of $\underline{P}_B$ ascertained, a series of perturbations (e.g. the passage of a TID) can be easily calculated from (5).

Some Alternative Equations

Except in the case of long perturbation scales, the above procedure can require considerable computation. The main problem arises from the need to recalculate Φ_A along the path for every dipole A position on the path.

An alternative approach that reduces this computation can be derived from a generalisation of equation (1)

$$\underline{E}_B \cdot \underline{J}_A = -\int_S \frac{2}{\eta} \underline{E}_B \cdot \underline{E}_A \, dS + j\omega \int_V \underline{E}_A \cdot \delta\varepsilon \cdot \underline{E}_B dV$$

Once again, the path between receiver and transmitter can be divided by a sequence of surfaces and the above equation used to develop the solution between them. In the limit that the length scales of the irregularities are large, the new propagation equation for the perturbations ρ_I will take the reduces to

$$\rho_I\left(s_{AB}\right) = \rho_I\left(s_{AB}\right) - \frac{j\beta}{2} \int_{s_S}^{s_{AB}} \left(I_J + \rho_J(s)\right) \hat{\varepsilon}_{IJ} \delta X_0 \frac{ds}{N_0} \qquad (6)$$

Once the background GO solution has been determined, the above equation can be used to obtain the effects of perturbations, such as the passage of a TID.